Exercises

Math tools

Exercise 1

Given :

$$u = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, v = \begin{bmatrix} 8 \\ 9 \\ 7 \end{bmatrix}, w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$Find$$
$$u \times v, u \times w, w \times v$$

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Solution

$$\vec{u} \times \vec{v} = -\vec{v} \times \vec{u} = \begin{bmatrix} u_y . v_z - u_z . v_y \\ u_z . v_x - u_x . v_z \\ u_x . v_y - u_y . v_x \end{bmatrix}$$

$$u \times v = \begin{bmatrix} -19 \\ 20 \\ -4 \end{bmatrix}, u \times w = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}, w \times v = \begin{bmatrix} -13 \\ 17 \\ -7 \end{bmatrix}$$

Exercise 2

Geiven

$$A = \begin{bmatrix} 4 & 5 & 6 \\ 1 & 4 & 7 \\ 3 & 2 & 9 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 9 \\ 5 & 8 & 7 \end{bmatrix}$$

$$Find$$
$$\det(A), A \times B, A^{-1}$$

Solution

$$\det(A) = 88$$

$$A \times B = \begin{bmatrix} 4 & 5 & 6 \\ 1 & 4 & 7 \\ 3 & 2 & 9 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 9 \\ 5 & 8 & 7 \end{bmatrix} = \begin{bmatrix} 43 & 90 & 103 \\ 41 & 83 & 89 \\ 53 & 93 & 93 \end{bmatrix}$$

$$A^{-1} = \frac{1}{88}\begin{bmatrix} 22 & -33 & 11 \\ 6 & 18 & -22 \\ -10 & 7 & 11 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{3}{8} & \frac{1}{8} \\ \frac{6}{44} & \frac{9}{44} & -\frac{1}{4} \\ -\frac{5}{44} & \frac{7}{88} & \frac{1}{8} \end{bmatrix} = \begin{bmatrix} 0.25 & -0.375 & 0.125 \\ 0.1364 & 0.2045 & -0.25 \\ -0.1136 & 0.0795 & 0.125 \end{bmatrix}$$

Exercise 3

Given the rotation matrices

$$R_0^1 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_1^2 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_2^3 = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Find\ R_0^3$$

Solution

$$R_0^2 = R_0^1 \times R_1^2 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{6}-\sqrt{2}}{4} & \frac{-\sqrt{6}-\sqrt{2}}{4} & 0 \\ \frac{\sqrt{6}+\sqrt{2}}{4} & \frac{\sqrt{6}-\sqrt{2}}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow R_0^2 = \begin{bmatrix} 0.2588 & -0.9659 & 0 \\ 0.9659 & 0.2588 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

$$R_0^3 = R_0^2 \times R_2^3 = \begin{bmatrix} \frac{\sqrt{6}-\sqrt{2}}{4} & \frac{-\sqrt{6}-\sqrt{2}}{4} & 0 \\ \frac{\sqrt{6}+\sqrt{2}}{4} & \frac{\sqrt{6}-\sqrt{2}}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow R_0^3 = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.7971 & -0.7071 & 0 \\ 0.7071 & -0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Exercise 4

- Given the rotation matrix

$$R_i^j = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}, R_k^l = \begin{bmatrix} 0.1464 & -0.8536 & 0.5 \\ 0.8536 & -0.1464 & -0.5 \\ 0.5 & 0.5 & 0.7971 \end{bmatrix}$$

$$Find\ (Euler\ and\ RPY)\ angles$$

Solution

$$R_i^j = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

Euler condition is satisfied:

$$\theta = \text{acos}(0) = \frac{\pi}{2}$$

$$\psi = atan2(1, -0) = \frac{\pi}{2}$$

$$\varphi = atan2(-1,0) = -\frac{\pi}{2}$$

RPY condition is not satisfied.

Solution

$$R_k^l = \begin{bmatrix} 0.1464 & -0.8536 & 0.5 \\ 0.8536 & -0.1464 & -0.5 \\ 0.5 & 0.5 & 0.7071 \end{bmatrix}$$

Euler condition is satisfied:

$$\theta = \mathrm{acos}(0.7071) = \frac{\pi}{4}$$

$$\psi = atan2(0.5,0.5) = \frac{\pi}{4}$$

$$\varphi = atan2(0.5,0.5) = \frac{\pi}{4}$$

Solution

RPY condition is satisfied:

$$\beta = -\operatorname{asin}(0.5) = -\frac{\pi}{6}$$

$$\alpha = atan2(0.8536, 0.1464) = 1.4009\ rad$$

$$\gamma = atan2(0.5, 0.7071) = 0.6155\ rad$$

Exercise 5

Given the transformation matrices:

$$T_0^1 = \begin{bmatrix} 0 & 1 & 0 & 10 \\ -1 & 0 & 0 & 15 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_1^2 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 25 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Find\ T_0^2, T_2^0$$

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Solution

$$T_0^2 = \begin{bmatrix} 0 & 1 & 0 & 10 \\ -1 & 0 & 0 & 15 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 25 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^2 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 35 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 15 \\ 0 & 0 & 1 & 30 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution

Remember that:

$$T_i^j = \begin{pmatrix} R_i^j & P_i^j \\ 0 & 1 \end{pmatrix} \Rightarrow$$

$$\left(T_i^j\right)^{-1} = T_j^i = \begin{pmatrix} {R_i^j}^T & -{R_i^j}^T \times P_i^j \\ 0 & 1 \end{pmatrix}$$

Solution

$$T_0^2 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 35 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 15 \\ 0 & 0 & 1 & 30 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -10\sqrt{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & -25\sqrt{2} \\ 0 & 0 & 1 & -30 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thanks