ROBOTICS

Direct and Inverse Velocity Model

DVM & IVM

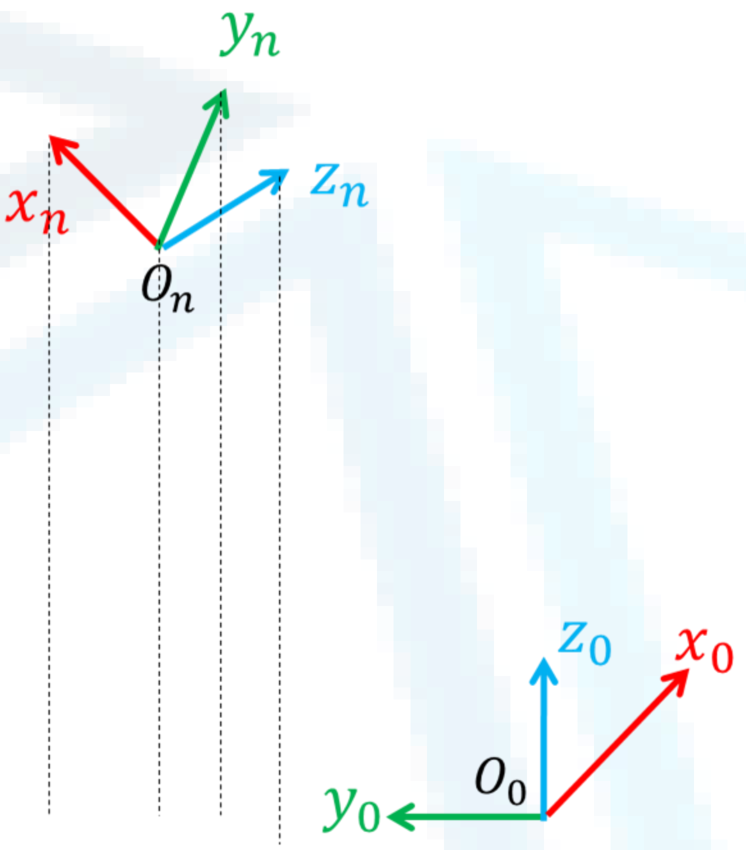
DVM

Consider a n-link robot :

w_0^n is the angular velocity of the end-effector

v_0^n is the linear velocity of the end-effector

$$\left.\begin{matrix} v_0^n = J_v\dot{q} \\ w_0^n = J_w\dot{q} \end{matrix}\right\} \left\{\begin{matrix} \xi = J\dot{q} \\ \xi = \begin{bmatrix} v_0^n \\ w_0^n \end{bmatrix} \\ J = \begin{bmatrix} J_v \\ J_w \end{bmatrix} \\ \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix} \end{matrix}\right.$$



The Jacobian

$$J_v = \begin{bmatrix} J_{v1} & J_{v2} & \cdots & J_{vn} \end{bmatrix} : J_{vi} = \begin{cases} z_{i-1} \times (o_n - o_{i-1}) & revolute-joint-i \\ z_{i-1} & prismatic-joint-i \end{cases}$$

$$J_w = \begin{bmatrix} J_{w1} & J_{w2} & \cdots & J_{wn} \end{bmatrix} : J_{wi} = \begin{cases} z_{i-1} & revolute-joint-i \\ 0 & prismatic-joint-i \end{cases}$$

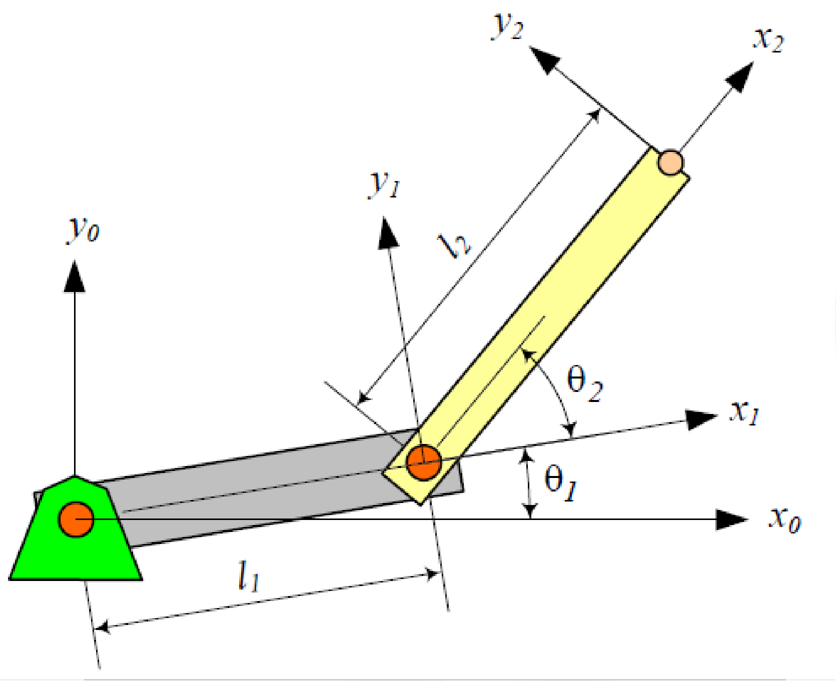
$$J = \begin{bmatrix} J_1 & J_2 & \cdots & J_n \end{bmatrix} : J_i = \begin{cases} \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix} & revolute-joint-i \\ \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} & prismatic-joint-i \end{cases}$$

Example: 2-link planer robot

$$J = \begin{bmatrix} z_0 \times (o_2 - o_0) & z_1 \times (o_2 - o_1) \\ z_0 & z_1 \end{bmatrix}$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, o_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}, o_2 = \begin{bmatrix} l_1 c_1 + l_2 c_{1,2} \\ l_1 s_1 + l_2 s_{1,2} \\ 0 \end{bmatrix}, z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{1,2} & -l_2 s_{1,2} \\ l_1 c_1 + l_2 c_{1,2} & l_2 c_{1,2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$



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IVM

$$\xi = J\dot{q} \Rightarrow \dot{q} = J^{-1}\xi$$

IVM

$Find : \dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3$

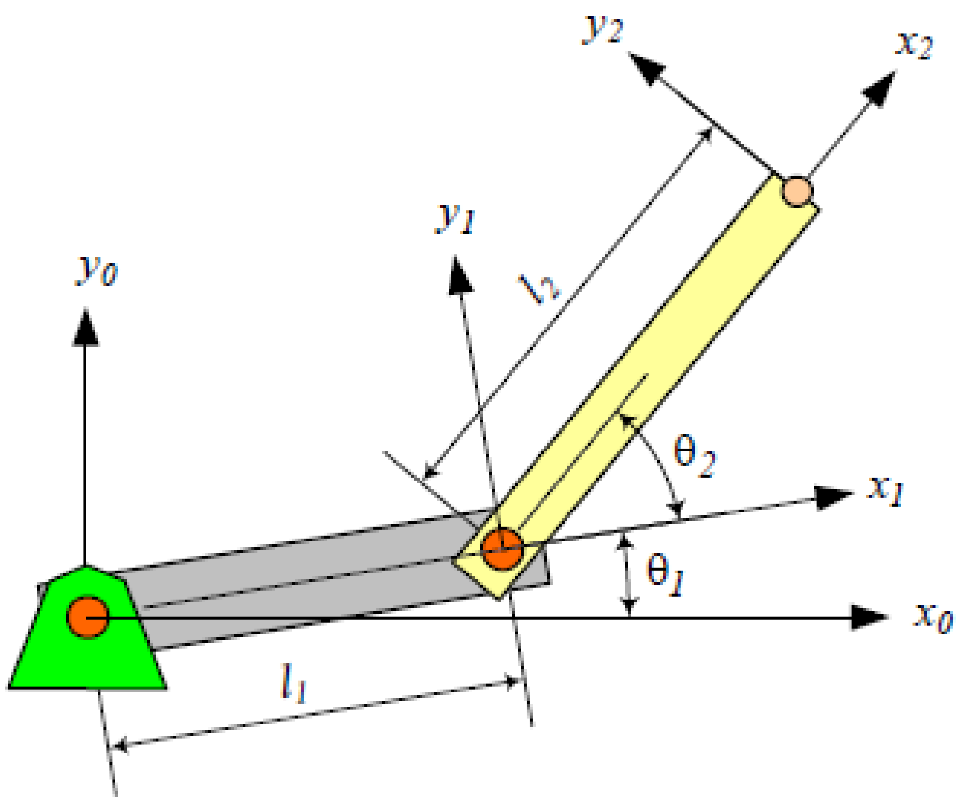
$When : \theta_1 \quad \theta_2 \quad \theta_3 \quad v_{O3} \quad w_{O3} \quad known$

2R Planer Robot

$$T_0^2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1c_1 + l_2c_{12} \\ s_{12} & c_{12} & 0 & l_1s_1 + l_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} l_1c_1 + l_2c_{12} \\ l_1s_1 + l_2s_{12} \\ 0 \end{bmatrix}, O_1 = \begin{bmatrix} l_1c_1 \\ l_1s_1 \\ 0 \end{bmatrix}$$

$$z_0 = z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



DVM & IVM of a 2R planer robot

- $\xi = \begin{bmatrix} v \\ w \end{bmatrix} = J\dot{q} = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$
- $v = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} -l_1 s_1 \dot{\theta}_1 - l_2 s_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ l_1 c_1 \dot{\theta}_1 + l_2 c_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} -l_1 s_1 \dot{\theta}_1 - l_2 s_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ l_1 c_1 \dot{\theta}_1 + l_2 c_{12}(\dot{\theta}_1 + \dot{\theta}_2) \end{bmatrix}$
- $w = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$

IVM of 2R planer robot

$$\begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix}=\begin{bmatrix}-l_1s_1-l_2s_{12} & -l_2s_{12}\\ l_1c_1+l_2c_{12} & l_2c_{12}\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}\Rightarrow\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}=\begin{bmatrix}-l_1s_1-l_2s_{12} & -l_2s_{12}\\ l_1c_1+l_2c_{12} & l_2c_{12}\end{bmatrix}^{-1}\begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix}$$

$$T=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\Rightarrow T^{-1}=\frac{-1}{\det(T)}\begin{bmatrix}-d & b\\ c & -a\end{bmatrix}$$

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}=\frac{-1}{l_1l_2s_2}\begin{bmatrix}-l_2c_{12} & -l_2s_{12}\\ l_1c_1+l_2c_{12} & l_1s_1+l_2s_{12}\end{bmatrix}^{-1}\begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix}$$

$$\dot{\theta}_1=\frac{l_2c_{12}\dot{X}+l_2s_{12}\dot{Y}}{l_1l_2s_2},\dot{\theta}_2=-\frac{(l_1c_1+l_2c_{12})\dot{X}+(l_1s_1+l_2s_{12})\dot{Y}}{l_1l_2s_2}$$

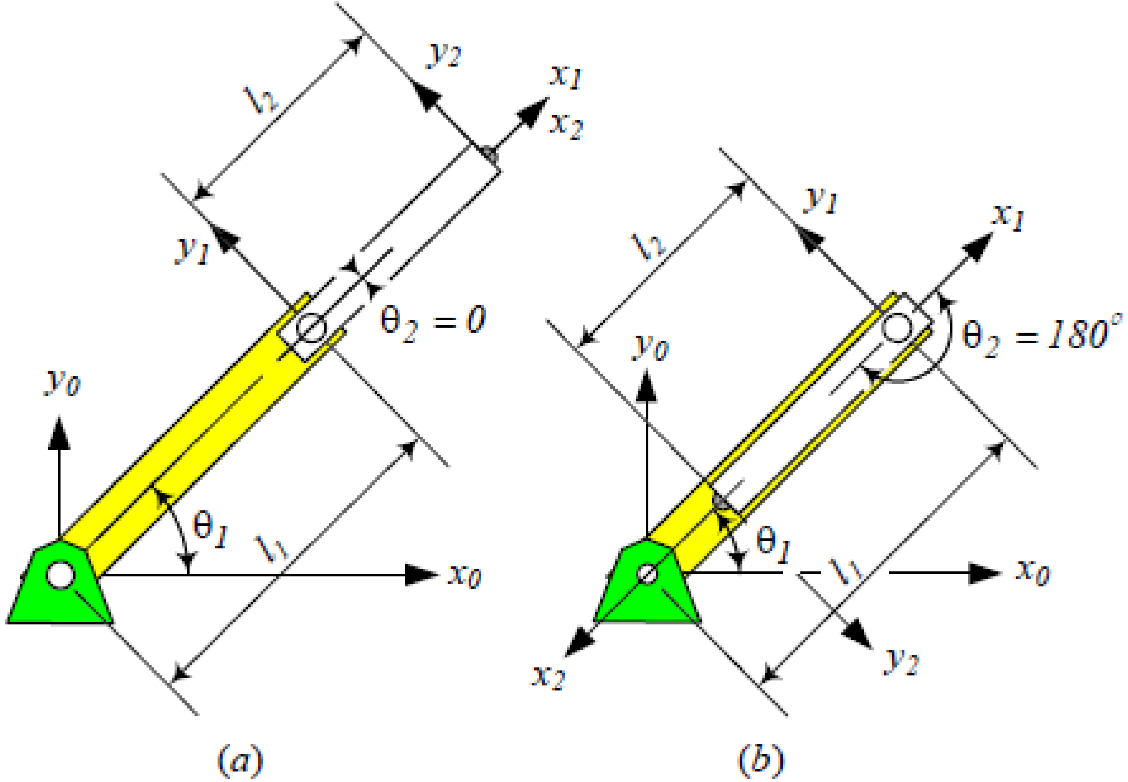
Singularity

$\det(J) = 0$

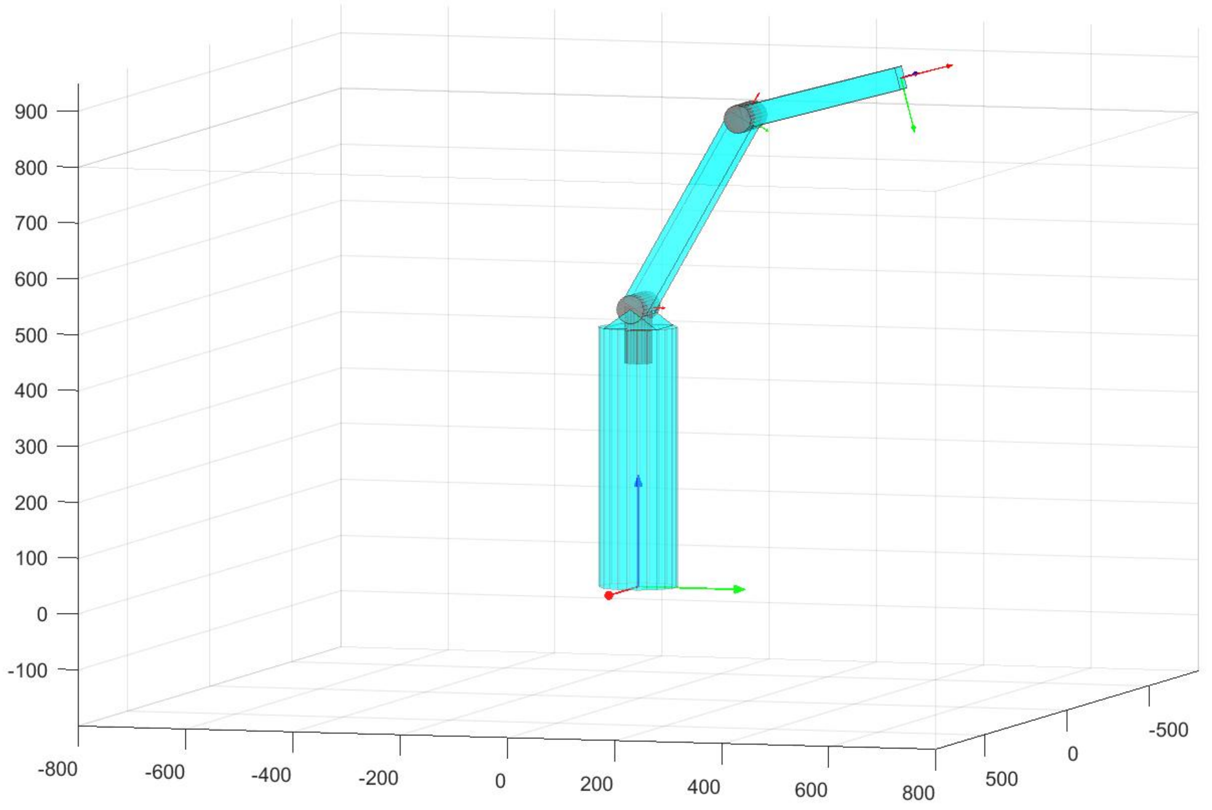
$2R\_Planer\_Robot$

$$|J| = 0 \Rightarrow l_1 l_2 s_2 = 0 \Rightarrow \begin{cases} \theta_2 = 0 \\ \theta_2 = \pi \end{cases}$$

$$\Rightarrow \begin{cases} (a) one\_motion\_direction \\ (b) \theta_1 = any\_value \end{cases}$$



Elbow manipulator



DH Table & Matrix

Link	a	α	d	θ
1	0	-90	500	Θ1+90
2	400	0	0	Θ2
3	300	0	0	Θ3

$$A_i = T_{i-1}^{i} = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation matrices

```
T01 =

[ -S1,   0, -C1,   0]
[  C1,   0, -S1,   0]
[   0,  -1,   0, 500]
[   0,   0,   0,   1]

T02 =

[ -C2*S1,   S1*S2, -C1,    -400*C2*S1]
[  C1*C2, -C1*S2, -S1,     400*C1*C2]
[    -S2,    -C2,   0, 500 - 400*S2]
[      0,      0,   0,             1]

T03 =

[ -C23*S1,   S1*S23, -C1, -100*S1*(4*C2 + 3*C23)]
[  C1*C23, -C1*S23, -S1,  100*C1*(4*C2 + 3*C23)]
[    -S23,    -C23,   0, 500 - 300*S23 - 400*S2]
[       0,       0,   0,                      1]
```

Vectors needed to calculate the Jacobean

```
Z1 =     O1 =     Z2 =     O2 =                   O3 =

  -C1        0     -C1      -400*C2*S1     -100*S1*(4*C2 + 3*C23)
  -S1        0     -S1       400*C1*C2      100*C1*(4*C2 + 3*C23)
    0      500       0    500 - 400*S2     500 - 300*S23 - 400*S2
```

Jacobean matrix

$$J = \begin{bmatrix} -100C_1(4C_2 + 3C_{23}) & 100S_1(4S_2 + 3S_{23}) & 300S_1S_{23} \\ -100S_1(4C_2 + 3C_{23}) & -100C_1(4S_2 + 3S_{23}) & -300C_1S_{23} \\ 0 & -100(4C_2 + 3C_{23}) & -300C_{23} \\ 0 & -C_1 & -C_1 \\ 0 & -S_1 & -S_1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\xi = \begin{bmatrix} v \\ w \end{bmatrix} = J\dot{q}$$

DVM Numerical Example

- Find the linear and angular velocity of the end effector when:

$$\theta_1 = \frac{\pi}{2}, \theta_2 = -\pi/3, \theta_3 = \pi/6$$

$$\dot{\theta}_1 = 0.5, \dot{\theta}_2 = 0.3, \dot{\theta}_3 = 0.2$$

Solution

$$J = \begin{bmatrix} 0 & -50(3+4\sqrt{3}) & -150 \\ -50(4+3\sqrt{3}) & 0 & 0 \\ 0 & -50(4+3\sqrt{3}) & -150\sqrt{3} \\ 0 & 0 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Velocity of the end effector

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} -75 - 60\sqrt{3} \\ -100 - 75\sqrt{3} \\ -60 - 75\sqrt{3} \end{bmatrix} mm/sec$$

$$\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} 0 \\ -0.5 \\ 0.5 \end{bmatrix} rad/sec$$

IVM Numerical example

- Find the joints velocity needed to make the linear velocity of the end effector

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} -75 - 40\sqrt{3} \\ -20 - 15\sqrt{3} \\ -40 - 75\sqrt{3} \end{bmatrix} mm/sec$$

when:

$$\theta_1 = \frac{\pi}{2}, \theta_2 = -\pi/3, \theta_3 = \pi/6$$

Joints velocity

$$solution = \begin{bmatrix} \dot{\theta}_1 = 0.1 \\ \dot{\theta}_2 = 0.2 \\ \dot{\theta}_3 = 0.3 \end{bmatrix} rad/sec$$

Thanks