

Valid and Invalid Arguments

Epp Section 1.3



An argument is a sequence of statements.

All statements but the final one are called assumptions or hypothesis.

The final statement is called the conclusion.

An argument is valid if:

whenever all the assumptions are true, then the conclusion is true.

If today is Wednesday, then yesterday is Tuesday.

Today is Wednesday.

.: Yesterday is Tuesday.



طريقة الغقاكيد

If p then q.

p

. . q

If typhoon, then class cancelled. Typhoon.

· Class cancelled.

	assumptions conclusion			
р	q	p→q	р	q
Т	Т	Τ	T	T
Т	F	F	T	F
F	/ T	T	F	T
F	F	T	F	F



If p then q. ~q

If typhoon, then class cancelled. Class not cancelled.

.. No typhoon.

Modus tollens is Latin meaning "method of denying".



A student is trying to prove that propositions P, Q, and R are all true. She proceeds as follows.

First, she proves three facts:

- · P implies Q
- · Q implies R
- · R implies P.

Then she concludes,

``Thus P, Q, and R are all true.''

Proposed argument:

$$(P \rightarrow Q), (Q \rightarrow R), (R \rightarrow P)$$

assumption

Is it valid?

 $P \wedge Q \wedge R$

Dr. Iyad Hatem https://manara.edu.sy/

conclusion



Valid Argument?
$$(P \rightarrow Q), \ (Q \rightarrow R), \ (R \rightarrow P)$$

Is it valid?

$P \wedge Q \wedge R$

assumptions

Р	Q	R
Т	Т	Т
Т	Т	F
Т	F	Т
Т	F	F
F	Т	Т
F	Т	F
F	F	Т
F	F	F

$P \to Q$	$Q \to R$	$R \to P$
Т	Т	Т
Т	F	Т
F	Т	Т
F	Т	Т
Т	Т	F
T	F	Т
Т	Т	F
Т	Т	Т

conclusion

$P \wedge Q \wedge R$	OK?
T	yes
F	no

To prove an argument is not valid, we just need to find a counterexample.



 $\begin{array}{c|ccccc} p & q & p \rightarrow q & q \\ \hline T & T & T & T \\ \hline T & F & F & F \end{array}$

If p then q. q ∴ p

Assumptions are true, but not the conclusion.

If you are a fish, then you drink water. You drink water.

You are a fish.

conclusion



assumptions conclusion

If p then q. ∼p ∴ ~q

		,		
р	q	p→q	~p	~q
T	T	T	F	F
T	F	F	F	T
F	T	T	/ T \	F
F	F	T	Т	T

If you are a fish, then you drink water. You are not a fish. You do not drink water.

Modus Ponens example

- Assume you are given the following two statements:
 - "you are in this class"
 - "if you are in this class, you will get a grade" $p \rightarrow q$

:. *q*

- Let p = "you are in this class"
- Let q = "you will get a grade"
- By Modus Ponens, you can conclude that you will get a grade



Consider (p ∧ (p→q)) → q

р	q	$p \rightarrow q$	$p \land (p \rightarrow q))$	$(b \lor (b \rightarrow d)) \rightarrow d$
Т	Т	Т	Т	Т
Т	F	F	F	Т
F	Т	Т	F	T
F	F	T	F	Т

$$\begin{array}{c}
p \\
\underline{p \to q} \\
q
\end{array}$$

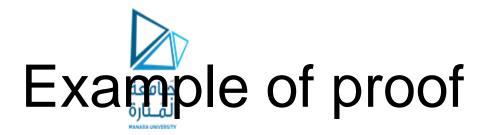
Generalization & Specialization

 Generalization: If you know that p is true, then p v q will ALWAYS be true

$$p \longrightarrow p \lor q$$

Specialization: If p \(\)
 q is true, then p will
 ALWAYS be true

$$\frac{p \wedge q}{p}$$



- We have the hypotheses:
- "It is not sunny this afternoon and it is colder than yesterday"
- "We will go swimming only if it is sunny"
- "If we do not go swimming, then we will take a canoe trip"
 - "If we take a canoe trip, then we will be home by sunset"
 - Does this imply that "we will be home by sunset"?

$$\neg p \land q$$

$$r \rightarrow p$$

$$\neg r \rightarrow s$$

$$s \rightarrow t$$

t

Example of proof

1st hypothesis

Simplification using step 1

3.
$$r \rightarrow p$$

2nd hypothesis

Modus tollens using steps 2 & 3

5.
$$\neg r \rightarrow s$$

3rd hypothesis

Modus ponens using steps 4 & 5

7.
$$s \rightarrow t$$

4th hypothesis

8. t

Modus ponens using steps 6 & 7

$$p \wedge q$$

$$n \rightarrow c$$

$$p \rightarrow q$$



We showed that:

$$-\left[\left(\neg p \land q\right) \land \left(r \to p\right) \land \left(\neg r \to s\right) \land \left(s \to t\right)\right] \to t$$

- That when the 4 hypotheses are true, then the implication is true
- In other words, we showed the above is a tautology!
- To show this, enter the following into the truth table generator at

http://sciris.shu.edu/~borowski/Truth/:

$$((\sim P \land Q) \land (R => P) \land (\sim R => S) \land (S => T)) => T$$

More rules of inference

 Conjunction: if p and q are true separately, then p\q is true p q $\therefore p \land q$

 Elimination: If pvq is true, and p is false, then q must be true $p \vee q$ $\neg p$

 Transitivity: If p→q is true, and q→r is true, then p→r must be true

$$p \to q$$

$$q \to r$$

$$r \to r$$

Even more rules of inference

- Proof by division into cases:
 if at least one of p or q is
 true, then r must be true
- $p \to r$ $q \to r$

:. r

 Contradiction rule: If ¬p→c is true, we can conclude p (via the contra-positive)

$$\frac{\neg p \to c}{p}$$

 Resolution: If pvq is true, and ¬pvr is true, then qvr must be true

$$p \lor q$$

$$\frac{\neg p \lor r}{\neg q \lor r}$$

$$\therefore q \lor r$$



Given the hypotheses:

- "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on"
- "If the sailing race is held, then the trophy will be awarded"
- "The trophy was not awarded"
- Can you conclude: "It rained"?

$$(\neg r \lor \neg f) \rightarrow (s \land l)$$

$$s \rightarrow t$$

r

Example of proof

2.
$$s \rightarrow t$$
 2nd hypothesis

4.
$$(\neg r \lor \neg f) \rightarrow (s \land I)$$
 1st hypothesis

5.
$$\neg (s \land I) \rightarrow \neg (\neg r \lor \neg f)$$
 Contrapositive of step 4

6.
$$(\neg s \lor \neg I) \rightarrow (r \land f)$$
 DeMorgan's law and double negation law

∴
$$p$$
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Consider the following:

q

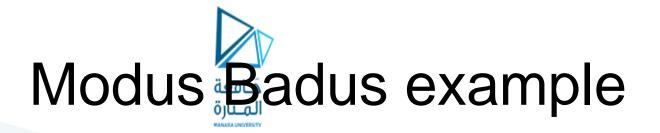
$$p \rightarrow q$$

$$\neg q \rightarrow \neg p$$

Is this true?

p	q	p→q	$d \lor (b \rightarrow d))$	$(d\lor(b\rightarrow d))\rightarrow b$
T	Т	Т	Т	Т
Т	F	F	F	T
F	Т	T	T	F
F	F	T	F	Т

Not a valid rule!



- Assume you are given the following two statements:
 - "you will get a grade"
 - "if you are in this class, you will get a grade"

$$p \to q$$

p.

- Let p = "you are in this class"
- Let q = "you will get a grade"

- You CANNOT conclude that you are in this class
 - You could be getting a grade for another class



Consider the following: ¬p

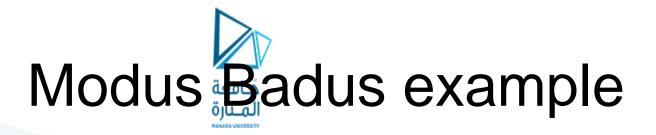
$$p \rightarrow q$$

• Is this true?

$$\therefore \neg q$$

р	q	p→q	$\neg p \land (p \rightarrow q))$	$(\neg p \land (p \rightarrow q)) \rightarrow \neg q$
Т	Т	Т	F	Т
Т	F	F	F	T
F	Т	Т	T	F
F	F	T	T	Т

Not a valid rule!



- Assume you are given the following two statements:
 - "you are not in this class"

 $p \rightarrow q$

- "if you are in this class, you will get a grade"

 $\therefore \neg q$

- Let p = "you are in this class"
- Let q = "you will get a grade"
- You CANNOT conclude that you will not get a grade
 - You could be getting a grade for another class