

Exercises 2: Matrices

CECC122: Linear Algebra and Matrix Theory

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Find, if possible, (a) $A + B$, (b) $A - B$, (c) $2A$, (d) $2A - B$, and (e) $B + 1/2A$

① $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix}$

(a) $A + B = \begin{bmatrix} -2 & 0 \\ 6 & 3 \end{bmatrix}$ (b) $A - B = \begin{bmatrix} 4 & 4 \\ -2 & -1 \end{bmatrix}$ (c) $2A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$

(d) $2A - B = \begin{bmatrix} 5 & 6 \\ 0 & 0 \end{bmatrix}$ (e) $B + \frac{1}{2}A = \begin{bmatrix} -5/2 & -1 \\ 5 & 5/2 \end{bmatrix}$

② $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix}$

$$(a) A + B = \begin{bmatrix} 4 & -2 & 5 \\ -4 & 0 & 2 \end{bmatrix}$$

$$(b) A - B = \begin{bmatrix} 0 & 4 & -3 \\ 2 & -2 & 6 \end{bmatrix}$$

$$(c) 2A = \begin{bmatrix} 4 & 2 & 2 \\ -2 & -2 & 8 \end{bmatrix}$$

$$(d) 2A - B = \begin{bmatrix} 2 & 5 & -2 \\ 1 & -3 & 10 \end{bmatrix}$$

$$(e) B + \frac{1}{2}A = \begin{bmatrix} 3 & -5/2 & 9/2 \\ -7/2 & 1/2 & 0 \end{bmatrix}$$

$$\textcircled{3} \quad A = \begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix}$$

$$(c) 2A = \begin{bmatrix} 12 & 0 & 6 \\ -2 & -8 & 0 \end{bmatrix}$$

All other operations are not defined (A and B are different sizes)

Find, if possible, (a) AB , (b) BA

① $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}$

$$(a) AB = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(-1) & 1(-1) + 2(8) \\ 4(2) + 2(-1) & 4(-1) + 2(8) \end{bmatrix} = \begin{bmatrix} 0 & 15 \\ 6 & 12 \end{bmatrix}$$

$$(b) BA = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2(1) + (-1)(4) & 2(2) + (-1)(2) \\ -1(1) + 8(4) & -1(2) + 8(2) \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 31 & 14 \end{bmatrix}$$

② $A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{bmatrix}$

(a) AB is not defined because A is 3×2 and B is 3×3

$$(b) BA = \begin{bmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0(2) + (-1)(-3) + 0(1) & 0(1) + (-1)(4) + 0(6) \\ 4(2) + 0(-3) + 2(1) & 4(1) + 0(4) + 2(6) \\ 8(2) + (-1)(-3) + 7(1) & 8(1) + (-1)(4) + 7(6) \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 10 & 16 \\ 26 & 46 \end{bmatrix}$$

$$\textcircled{3} \quad A = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$$(a) AB = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 3(2) + 2(3) + 1(0) \end{bmatrix} = \begin{bmatrix} 12 \end{bmatrix}$$

$$(b) AB = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(3) & 2(2) & 2(1) \\ 3(3) & 3(2) & 3(1) \\ 0(3) & 0(2) & 0(1) \end{bmatrix} = \begin{bmatrix} 6 & 4 & 2 \\ 9 & 6 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\textcircled{4} \quad A = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} -5 & 0 \\ 0 & 4 \end{bmatrix}$$

$$(a) AB = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} -5 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -10 & 0 \\ 0 & -12 \end{bmatrix}$$

$$(b) BA = \begin{bmatrix} -5 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} -10 & 0 \\ 0 & -12 \end{bmatrix}$$

⑤ $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

$$(a) AB = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 9 \\ 0 & -1 & 6 \\ 0 & 0 & 12 \end{bmatrix}$$

$$(b) BA = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 3 \\ 0 & -1 & 8 \\ 0 & 0 & 12 \end{bmatrix}$$

Write the system of linear equations in the form $Ax = b$ and solve this matrix equation for x

①

$$\begin{aligned}x + y &= 4 \\ -2x + y &= 0\end{aligned}$$

$$\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

Use Gauss-Jordan elimination on the augmented matrix

$$\begin{bmatrix} -1 & 1 & 4 \\ -2 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 8 \end{bmatrix}$$

Therefore, the solution is $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$$\textcircled{2} \quad \begin{aligned} 2x - y - z &= 0 \\ x - 2y + 2z &= 0 \end{aligned}$$

$$\begin{bmatrix} 2 & -1 & -1 \\ 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Use Gauss-Jordan elimination on the augmented matrix

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ 1 & -2 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -4/3 & 0 \\ 0 & 1 & -5/3 & 0 \end{bmatrix} \Rightarrow \begin{aligned} x - 4/3z &= 0 \\ y - 5/3z &= 0 \end{aligned}$$

Free variable: $x = t$

Therefore, the solution is
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ (5/4)t \\ (3/4)t \end{bmatrix} = t \begin{bmatrix} 1 \\ 5/4 \\ 3/4 \end{bmatrix}, t \in R$$

Solve for X in the equation, given

$$A = \begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}$$

(a) $3X + 2A = B$

(b) $2A - 5B = 3X$

(c) $X - 3A + 2B = O$ (d) $6X - 4A - 3B = O$

(a) $3X + 2A = B$

$$3X = B - 2A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix} - \begin{bmatrix} -8 & 0 \\ 2 & -10 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ -4 & 11 \\ 10 & 0 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 3 & 2/3 \\ -4/3 & 11/3 \\ 10/3 & 0 \end{bmatrix}$$

(b) $2A - 5B = 3X$

$$3X = 2A - 5B = \begin{bmatrix} -8 & 0 \\ 2 & -10 \\ -6 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 10 \\ -10 & 5 \\ 20 & 20 \end{bmatrix} = \begin{bmatrix} -13 & -10 \\ 12 & -15 \\ -26 & -16 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -13/3 & -10/3 \\ 4 & -5 \\ -26/3 & -16/3 \end{bmatrix}$$

(c) $X - 3A + 2B = O$

$$X = 3A - 2B = \begin{bmatrix} -12 & 0 \\ 3 & -15 \\ -9 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ -4 & 2 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} -14 & -4 \\ 7 & -17 \\ -17 & -2 \end{bmatrix}$$

(d) $6X - 4A - 3B = O$

$$6X = 4A + 3B = \begin{bmatrix} -16 & 0 \\ 4 & -20 \\ -12 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -6 & 3 \\ 12 & 12 \end{bmatrix} = \begin{bmatrix} -13 & 6 \\ -2 & -17 \\ 0 & 20 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -13/6 & 1 \\ -1/3 & -17/6 \\ 0 & 10/3 \end{bmatrix}$$

Consider the matrices below

$$X = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad Z = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \quad W = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

- (a) Find scalars a and b such that $Z = aX + bY$
- (b) Show that there do not exist scalars a and b such that $W = aX + bY$
- (c) Show that if $aX + bY + cW = O$, then $a = 0$, $b = 0$, and $c = 0$
- (d) Find scalars a , b , and c , not all equal to zero, such that $aX + bY + cZ = O$

$$(a) \quad aX + bY = a \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \Rightarrow \begin{array}{rcl} a + b & = & 2 \\ & b & = -1 \\ a & = & 3 \end{array}$$

The only solution to this system is: $a = 3$ and $b = -1$



$$(b) \quad aX + bY = a \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{array}{rcl} a + b & = & 1 \\ & b & = 1 \\ a & = & 1 \end{array}$$

The system is inconsistent. No values of a and b will satisfy the equation

$$(c) \quad aX + bY + cW = a \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{rcl} a + b + c & = & 0 \\ & b + c & = 0 \\ a & + & c = 0 \end{array}$$

Then $a = -c$, so $b = 0$. Then $c = 0$, so $a = b = c = 0$

$$(d) \quad aX + bY + cZ = a \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{rcl} a + b + 2c & = & 0 \\ & b - c & = 0 \\ a & + & 3c = 0 \end{array}$$

Using Gauss-Jordan elimination the solution is $a = -3t$, $b = t$ and $c = t$, where t is any real number. If $t = 1$, then $a = -3$, $b = 1$, and $c = 1$

Determine whether the matrix is symmetric, skew-symmetric, or neither

① $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$$A^T = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -A \quad \text{The matrix is skew-symmetric}$$

② $A = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 3 & 0 \end{bmatrix}$

$$A^T = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 3 & 0 \end{bmatrix} = A \quad \text{The matrix is symmetric}$$

**Show that B is the inverse
of A**

① $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

② $A = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$



$$AB = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix} \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find the inverse of the matrix (if it exists)

① $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

$$A^{-1} = \frac{1}{7-6} \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

$$\textcircled{2} \quad A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/3 \end{bmatrix}$$

$$\textcircled{3} \quad A = \begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix}$$

$\det(A) = 0 \Rightarrow A$ is singular and has no inverse

$$\textcircled{4} \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$$

$$[A : I] = \begin{bmatrix} 1 & 1 & 1 & : & 1 & 0 & 0 \\ 3 & 5 & 4 & : & 0 & 1 & 0 \\ 3 & 6 & 5 & : & 0 & 0 & 1 \end{bmatrix}$$

Using elementary row operations, rewrite this matrix in reduced row-echelon form

$$\left[I : A^{-1} \right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array} \right] \Rightarrow A^{-1} = \left[\begin{array}{ccc} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{array} \right]$$

⑤ $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ 7 & 16 & -21 \end{bmatrix}$

$$\left[A : I \right] = \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 7 & -10 & 0 & 1 & 0 \\ 7 & 16 & -21 & 0 & 0 & 1 \end{array} \right]$$

Using elementary row operations, you cannot form the identity matrix on the left side

$$\left[\begin{array}{ccc|cc} 1 & 0 & 13 & 0 & -16 & 7 \\ 0 & 1 & -7 & 0 & 7 & -3 \\ 0 & 0 & 0 & 1 & 2 & -1 \end{array} \right]$$

Therefore, the matrix is singular and has no inverse

$$\textcircled{6} \quad A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/5 \end{bmatrix}$$

Use an inverse matrix to solve each system of linear equations

①

$$\begin{aligned}x + 2y &= -1 \\x - 2y &= 3\end{aligned}$$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{-2-2} \begin{bmatrix} -2 & -2 \\ -1 & 1 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}$$

$$\mathbf{x} = A^{-1}\mathbf{b} = \frac{1}{4} \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

②

$$\begin{aligned}x + 2y + z &= 2 \\x + 2y - z &= 4 \\x - 2y + z &= -2\end{aligned}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & -1 \\ 1 & -2 & 1 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{4} \begin{bmatrix} 0 & 2 & 2 \\ 1 & 0 & -1 \\ 2 & -2 & 0 \end{bmatrix}$$

$$\mathbf{x} = A^{-1}\mathbf{b} = \frac{1}{4} \begin{bmatrix} 0 & 2 & 2 \\ 1 & 0 & -1 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

The solution is: $x = 1$, $y = 1$ and $z = -1$

(a) Find $2A - A^2$,

$$A = \begin{bmatrix} -1 & -2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Conclude A^{-1}

(a)

$$\begin{aligned}
 2A - A^2 &= 2 \begin{bmatrix} -1 & -2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & -2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} -2 & -4 & 0 \\ 4 & 6 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} -3 & -4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3
 \end{aligned}$$

(b)

$$2A - A^2 = A(2I - A) = I_3 \Rightarrow A^{-1} = (2I - A)$$

$$A^{-1} = 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & -2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 0 \\ -2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find, if possible, (a) $A + B$, (b) $A - B$, (c) $2A$, (d) $3A - B$, and (e) $2B + 1/3A$

$$\textcircled{1} \quad A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 \\ -2 & -3 \end{bmatrix}$$

$$\textcircled{2} \quad A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & -2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\textcircled{3} \quad A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -2 & 4 \\ -3 & 1 & 3 \end{bmatrix}$$

Find, if possible, (a) AB , (b) BA

$$\textcircled{1} \quad A = \begin{bmatrix} 2 & -2 \\ -1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 1 \\ 2 & -2 \end{bmatrix}$$

$$\textcircled{2} \quad A = \begin{bmatrix} 3 & 2 & 1 \\ -3 & 0 & 4 \\ 4 & -2 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$$

$$\textcircled{3} \quad A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2 & 4 \end{bmatrix}$$

$$\textcircled{4} \quad A = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad B = [1 \ 2 \ 1]$$

Write the system of linear equations in the form $Ax = b$ and solve this matrix equation for

①
$$\begin{aligned} 2x + 3y &= 5 \\ x + 4y &= 10 \end{aligned}$$

②
$$\begin{aligned} x + y - 3z &= -1 \\ -x + 2y &= 1 \\ 2x - y + z &= 2 \end{aligned}$$

Find the inverse of the matrix (if it exists)

①
$$A = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

②
$$A = \begin{bmatrix} -7 & 33 \\ 4 & -19 \end{bmatrix}$$

③
$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$

④
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 2 & 5 & 5 \end{bmatrix}$$

Use an inverse matrix to solve each system of linear equations

$$\textcircled{1} \begin{cases} 2x - y = -3 \\ 2x + y = 7 \end{cases}$$

$$\textcircled{2} \begin{cases} x + y - 2z = 0 \\ x - 2y + z = 0 \\ x - y - z = -1 \end{cases}$$

(a) Find $A^3 - 5A^2 + 8A$, (b) Conclude A^{-1}

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$

$$(A + B)^2 = A^2 + 2AB + B^2 ?$$

$$(A - B)^2 = A^2 - 2AB + B^2 ?$$

$$(A + B)(A - B) = A^2 - B^2 ?$$