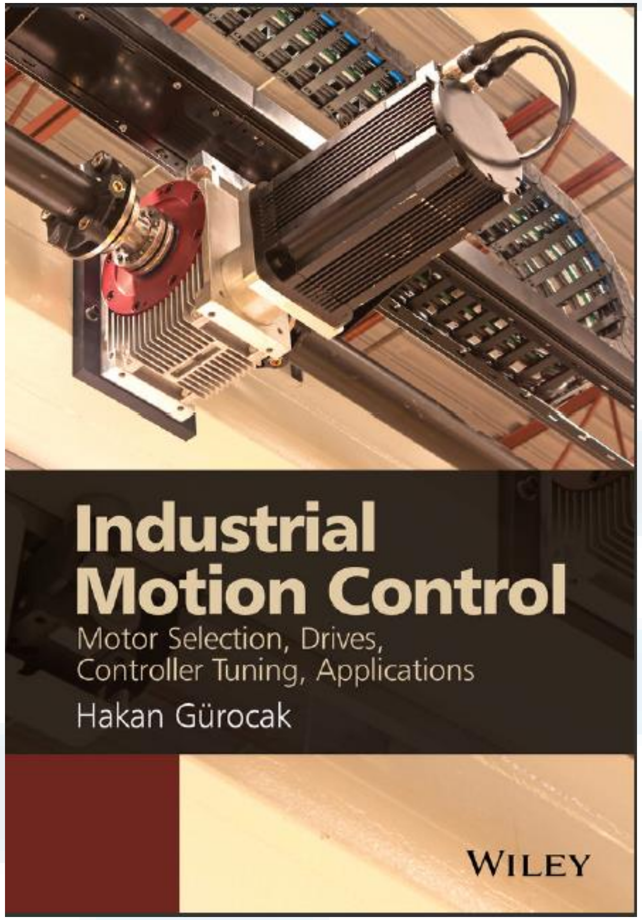
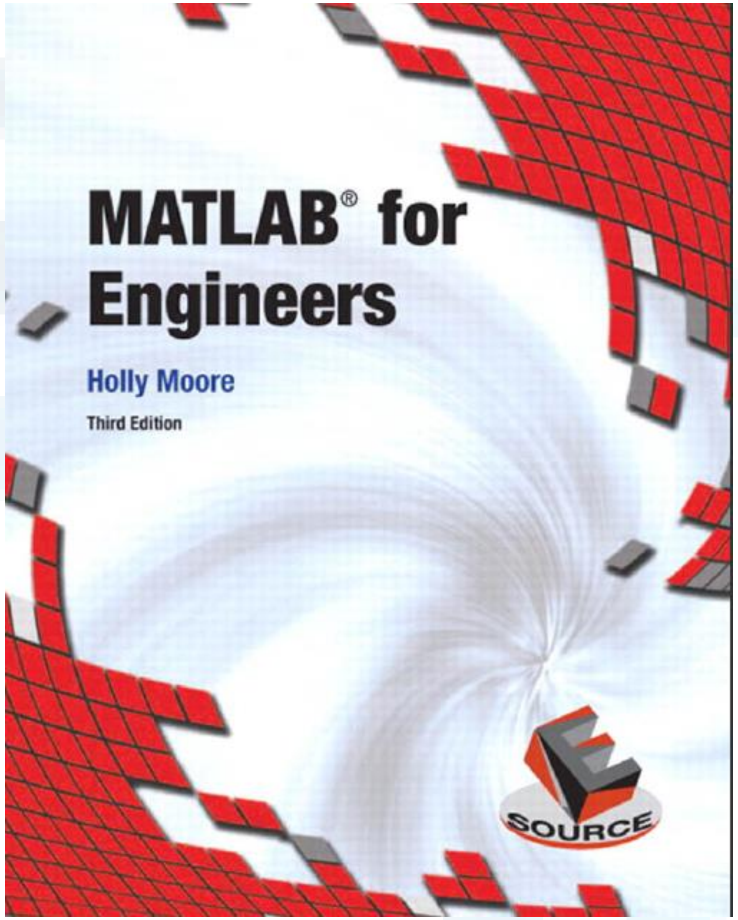
Robot Control

Introduction

Textbook & Reference





Syllabus

- Dynamic motion equations
  - Newton Euler
  - Lagrange
- Motor velocity, position and acceleration profile
  - Trapezoid velocity profile
  - S-curve velocity profile
- Multi-axis motion
  - Slew motion
  - Interpolated motion

Dynamic motion equations

Newton-Euler & Lagrange

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Dynamic Motion Equations

Newton - Euler

$$\sum F_{i/0} = m_i a_{i/0}$$

$$\sum M_{i/0} = I_{i/0} \alpha_{i/0}$$

$a_{i/0}$: linear acceleration of i in 0
$\propto_{i/0}$: angular acceleration of i in 0
$I_{i/0}$: mass moment of inertia matrix of i in 0

Lagrange

$$\mathcal{L} = K - V$$

$\mathcal{L}$: Lagrangean
K: Kinetic energy
V: Potential energy

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i \quad i = 1,2,\ldots n$$

Q_i: Generalized force of joint i

$$Q_i = M_i + J^T F_e$$

M_i: Actuator force
F_e: External force on the end-effector

$$F_e = \begin{bmatrix} -F_{en} \\ -M_{en} \end{bmatrix}$$

Example (Newton Euler)

- *Assume Or is the center of gravity*

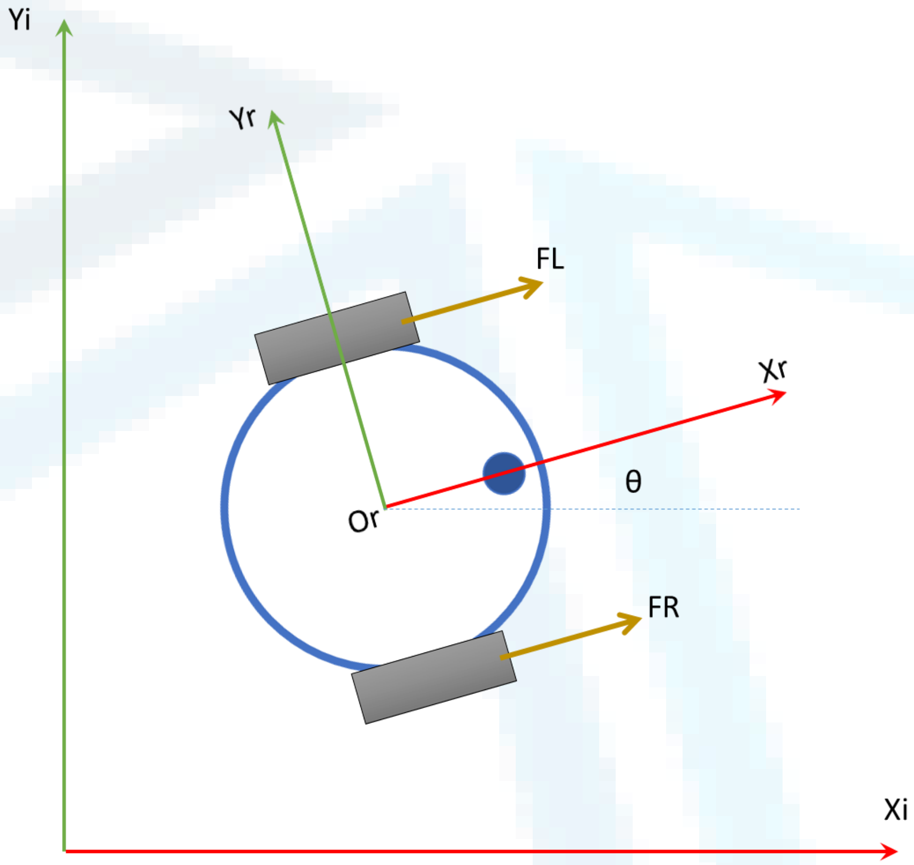
$$\sum F_{/Or} = m\dot{V}$$

$$\sum M_{/Or} = I\dot{\omega}$$

$$\tau_r = FR \times r, \tau_l = FL \times r$$

$$\sum F_{/Or} = FR + FL = \frac{1}{r}(\tau_r + \tau_l)$$

$$\sum M_{/Or} = d(FR - FL) = \frac{d}{r}(\tau_r - \tau_l)$$



Example (Lagrange)

- Find the dynamic motion equations if you know that the lagrangian of the robot is given as following:

$$\mathcal{L} = \frac{3}{2}\dot{\theta}_1^{\ 2} + \left(\dot{\theta}_1 + \dot{\theta}_2\right)^2 + \cos(\theta_2)(\dot{\theta}_1^{\ 2} + \dot{\theta}_1\dot{\theta}_2) - 20\sin(\theta_1) - 10\sin(\theta_1 + \theta_2)$$

- Write the answer in matrices form.

Solution

- $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_i}\right) - \frac{\partial \mathcal{L}}{\partial \theta_i} = \tau_i \text{ , } i = 1,2$
- $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1}\right) - \frac{\partial \mathcal{L}}{\partial \theta_1} = \tau_1$
- $\tau_1 = (5 + 2C_2)\ddot{\theta}_1 + (2 + C_2)\ddot{\theta}_2 - S_2\left(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2{}^2\right) + 20C_1 + 10C_{12}$
- $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}\right) - \frac{\partial \mathcal{L}}{\partial \theta_2} = \tau_2$
- $\tau_2 = (2 + C_2)\ddot{\theta}_1 + (2)\ddot{\theta}_2 + S_2\dot{\theta}_1{}^2 + 10C_{12}$

Matrices form

$$\begin{bmatrix} 5+2C_2 & 2+C_2 \\ 2+C_2 & 2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -S_2\left(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2{}^2\right) \\ S_2\dot{\theta}_1{}^2 \end{bmatrix} + \begin{bmatrix} 20C_1 + 10C_{12} \\ 10C_{12} \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

$$D_{(\theta)} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + H_{(\theta,\dot{\theta})} + G_{(\theta)} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

Thanks

What is the best method to generate the dynamic equations?