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Robot Control

Exercises

Exercise 1

- Find the dynamic motion equations of a 2 DOF planar robot, if you know the lagrangian:

$$\mathcal{L} = K - P$$

$$\mathcal{L} = k_1 \dot{\theta_1}^2 + (k_2 + k_3 \cos(\theta_2))(\dot{\theta_1} + \dot{\theta_2})^2 - p_1 \sin(\theta_1) - p_2 \sin(\theta_1 + \theta_2)$$

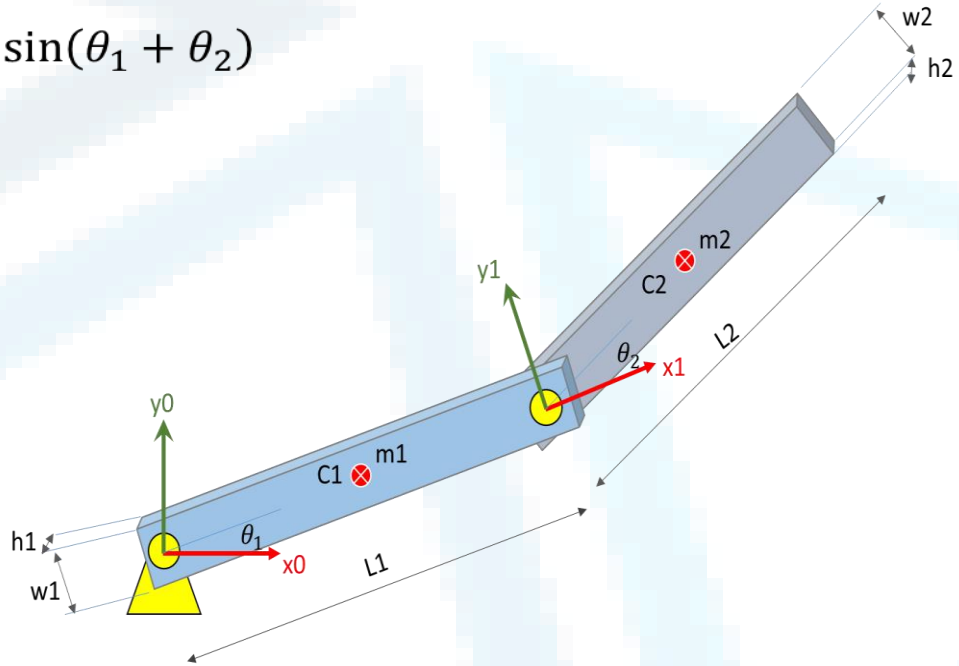
$$k_1 = \frac{m_1}{24}(7{L_1}^2 + {w_1}^2)$$

$$k_2 = \frac{m_1 + m_2}{2}{L_1}^2 + \frac{7m_2}{24}{L_2}^2 + \frac{m_2}{24}{w_2}^2$$

$$k_3 = \frac{m_2}{2} L_1 L_2$$

$$p_1 = g.L_1\left(\frac{m_1}{2} + m_2\right)$$

$$p_2 = g.L_2.\frac{m_2}{2}$$



Solution

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = \tau_i \, , i = 1,2$$

$$\begin{bmatrix} 2(k_1 + k_2 + k_3 C_2) & 2(k_2 + k_3 C_2) \\ 2(k_2 + k_3 C_2) & 2(k_2 + k_3 C_2) \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -2k_3 S_2 \left(\dot{\theta}_1 \dot{\theta}_2 + {\dot{\theta}_2}^2\right) \\ k_3 S_2 \left({\dot{\theta}_1}^2 - {\dot{\theta}_2}^2\right) \end{bmatrix} + \begin{bmatrix} p_1 C_1 + p_2 C_{12} \\ p_2 C_{12} \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

$$D(q)\ddot{q} + H(q, \dot{q}) + G(q) = \mathrm{Q}$$

- $D(q)\ddot{q}$: The Acceleration-related Inertia matrix term, Symmetric
- $H(q, \dot{q})$: The Coriolis and Centrifugal terms
- $G(q)$: The Gravity terms
- Q: Driving torque applied on each link

Exercise 2

Given the lagrangian of a two DOF robot:

$$\mathcal{L} = 0.1\dot{\theta_1}^2 + (0.2 + 0.05\cos(\theta_2))\left(\dot{\theta_1} + \dot{\theta_2}\right)^2 - 5\sin(\theta_1) - 0.6\sin(\theta_1 + \theta_2)$$

Find the dynamic motion equations of the robot.

Solution

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = \tau_i \, , i = 1,2$$

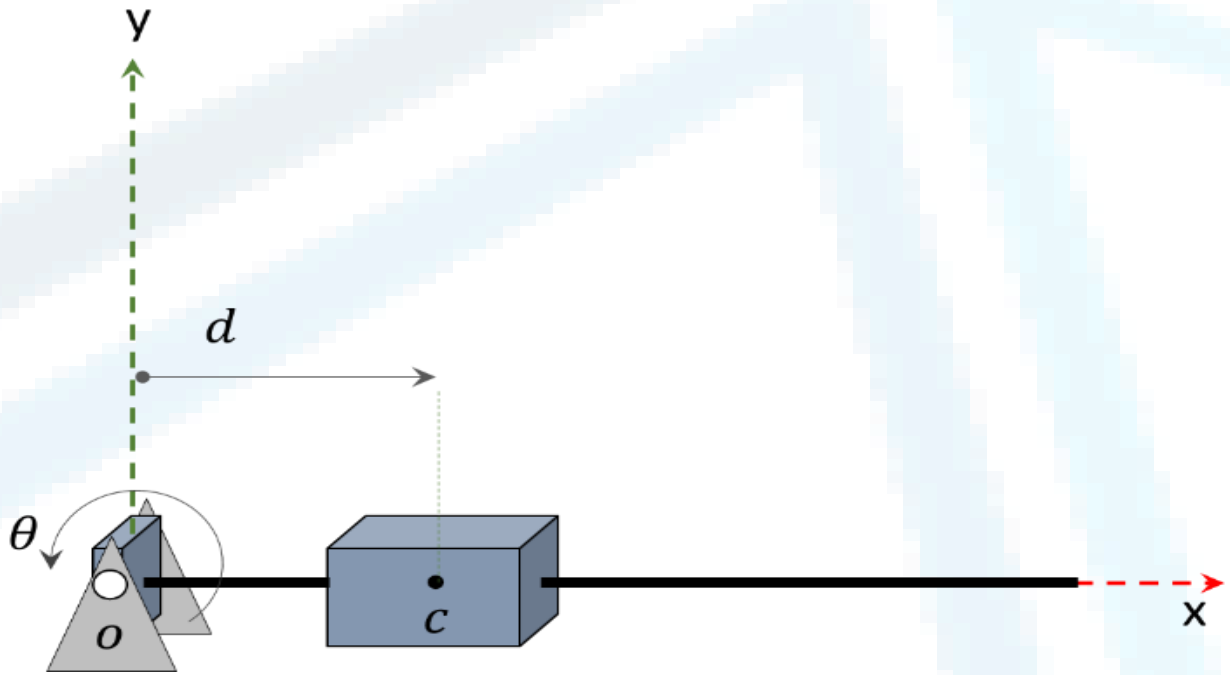
$$\begin{bmatrix} 0.6 + 0.1C_2 & 0.4 + 0.1C_2 \\ 0.4 + 0.1C_2 & 0.4 + 0.1C_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -0.1S_2\left(\dot{\theta}_1\dot{\theta}_2 + {\dot{\theta}_2}^2\right) \\ 0.05S_2\left({\dot{\theta}_1}^2 - {\dot{\theta}_2}^2\right) \end{bmatrix} + \begin{bmatrix} 5C_1 + 0.6C_{12} \\ 0.6C_{12} \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

Homework

Given the lagrangian of a RP polar robot as following:

$$\mathcal{L} = (0.05 + 10d^2)\dot{\theta}^2 + 5\dot{d}^2 + 100 \times d \times \sin(\theta)$$

Find the dynamic motion equations.



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Thanks