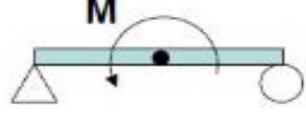
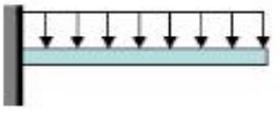
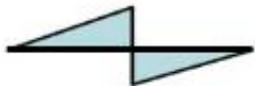


المحاضرة الخامسة

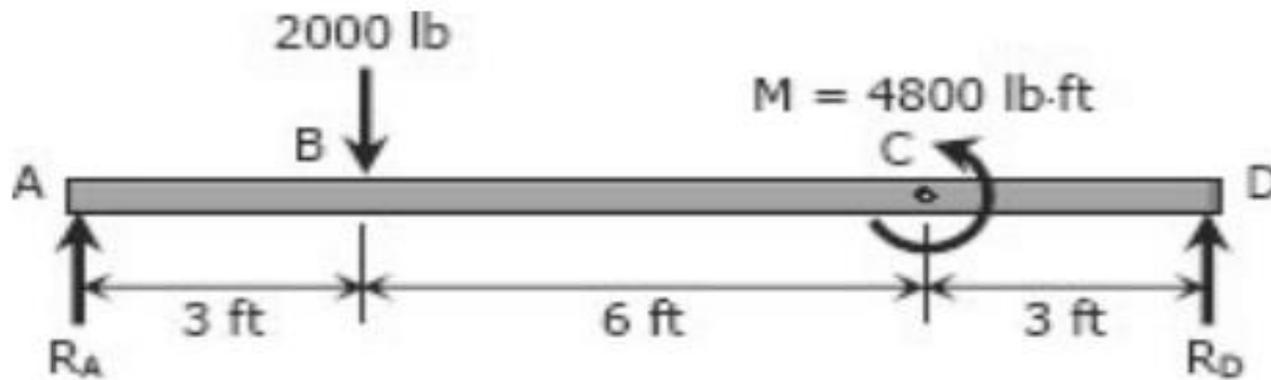
مخططات قوى القص وعزوم الانحناء (2)

د. نزار عبد الرحمن

Common Relationships

Load	0 	0 	Constant 
Shear	Constant 	Constant 	Linear 
Moment	Linear 	Linear 	Parabolic 

- **مسألة (1):** ارسم مخططات قوى القص وعزوم الانحناء للعتبة المبينة في الشكل .
(افرض القوى بالنيوتن والأبعاد بالمتر).



حساب ردود الأفعال

$$\begin{aligned}\sum M_A &= 0 \\ 12R_D + 4800 &= 3(2000) \\ R_D &= 100 \text{ lb}\end{aligned}$$

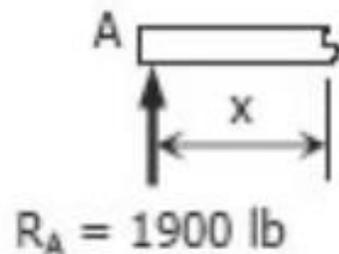
$$\begin{aligned}\sum M_D &= 0 \\ 12R_A &= 9(2000) + 4800 \\ R_A &= 1900 \text{ lb}\end{aligned}$$



Segment AB:

$$V_{AB} = 1900 \text{ lb}$$

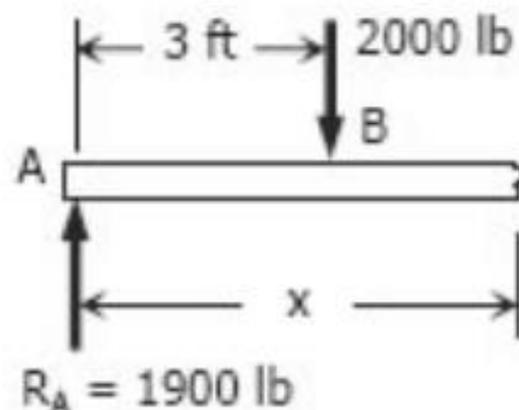
$$M_{AB} = 1900x \text{ lb}\cdot\text{ft}$$

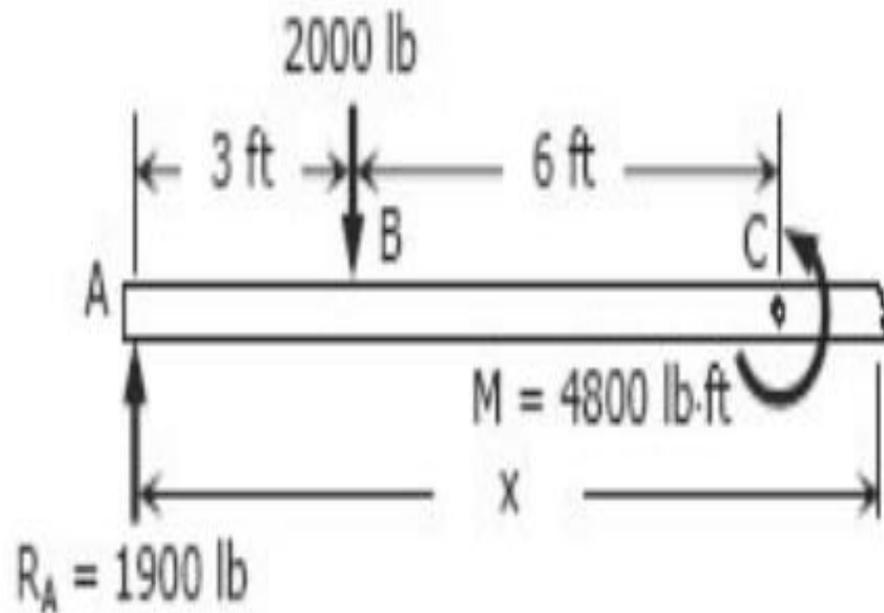


Segment BC:

$$\begin{aligned} V_{BC} &= 1900 - 2000 \\ &= -100 \text{ lb} \end{aligned}$$

$$\begin{aligned} M_{BC} &= 1900x - 2000(x - 3) \\ &= 1900x - 2000x + 6000 \\ &= -100x + 6000 \end{aligned}$$





Segment CD:

$$\begin{aligned}V_{CD} &= 1900 - 2000 \\ &= -100 \text{ lb}\end{aligned}$$

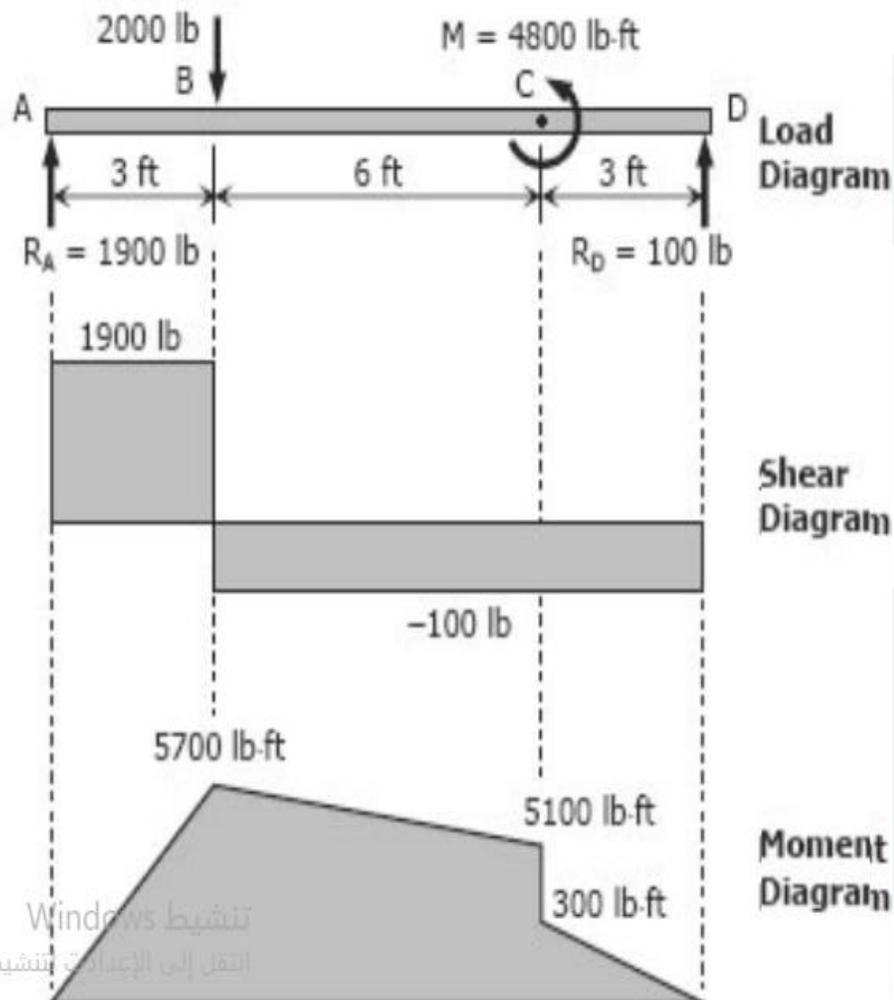
$$\begin{aligned}M_{CD} &= 1900x - 2000(x - 3) - 4800 \\ &= 1900x - 2000x + 6000 - 4800 \\ &= -100x + 1200\end{aligned}$$



=

a- رسم مخطط قوى القص :
1- في الجزء AB يكون توزيع قوى القص منتظما عند القيمة 1900 نيوتن.

2- في الجزئين و BC و BD يكون توزيع قوة القص منتظما عند القيمة 100 نيوتن



b- رسم مخطط عزم الانحناء :

1- من أجل الجزء AB، $M_{AB} = 1900X$ ، معادلة خطية .

- عند $M_{AB} = 0, X = 0$

- عند $M_{AB} = 5700 \text{ N.M}, X = 3 \text{ m}$

2- من أجل الجزء BC:

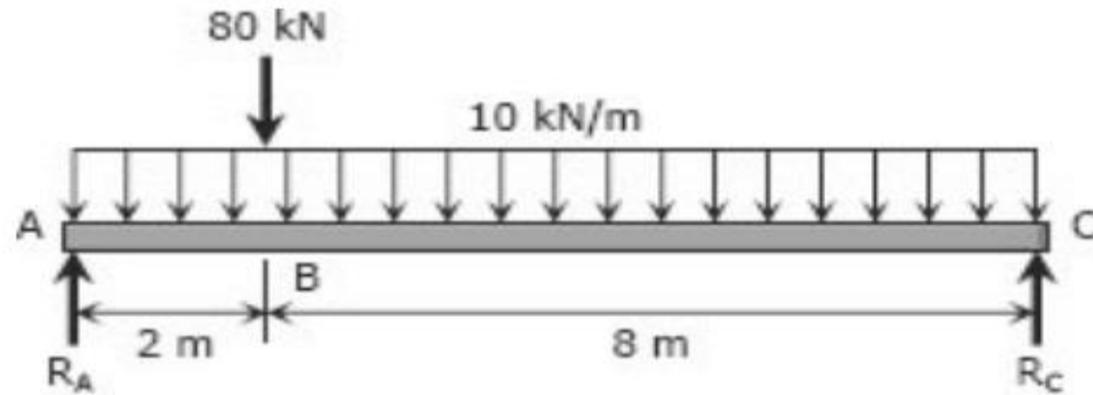
المعادلة $M_{BC} = 100X + 6000$ معادلة خطية

- عند $M_{BC} = 5700 \text{ N.M}, X = 3 \text{ m}$

- عند $M_{BC} = 5100 \text{ N.M}, X = 9 \text{ m}$

- من أجل الجزء CD: المعادلة خطية أيضا
- $M_{CD} = -100X + 1200$
- عند $X = 9\text{m}$, $M_{CD} = 300\text{N.M}$
- عند $X = 12\text{m}$, $M_{CD} = 0$

مسألة (2): ارسم مخططات قوى القص وعزوم الانحناء للعتبة المبينة في الشكل .



حساب ردود الأفعال :

$$\sum M_A = 0$$

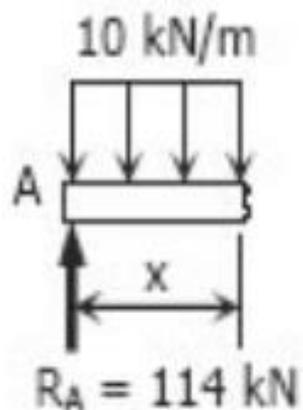
$$10R_C = 2(80) + 5[10(10)]$$

$$R_C = 66 \text{ kN}$$

$$\sum M_C = 0$$

$$10R_A = 8(80) + 5[10(10)]$$

$$R_A = 114 \text{ kN}$$



Segment AB:

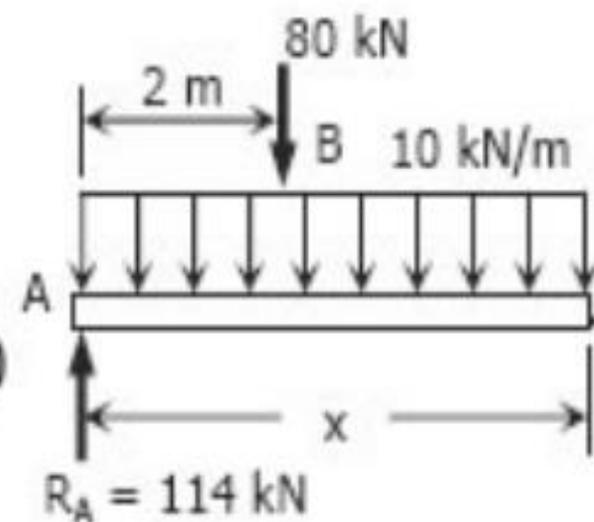
$$V_{AB} = 114 - 10x \text{ kN}$$

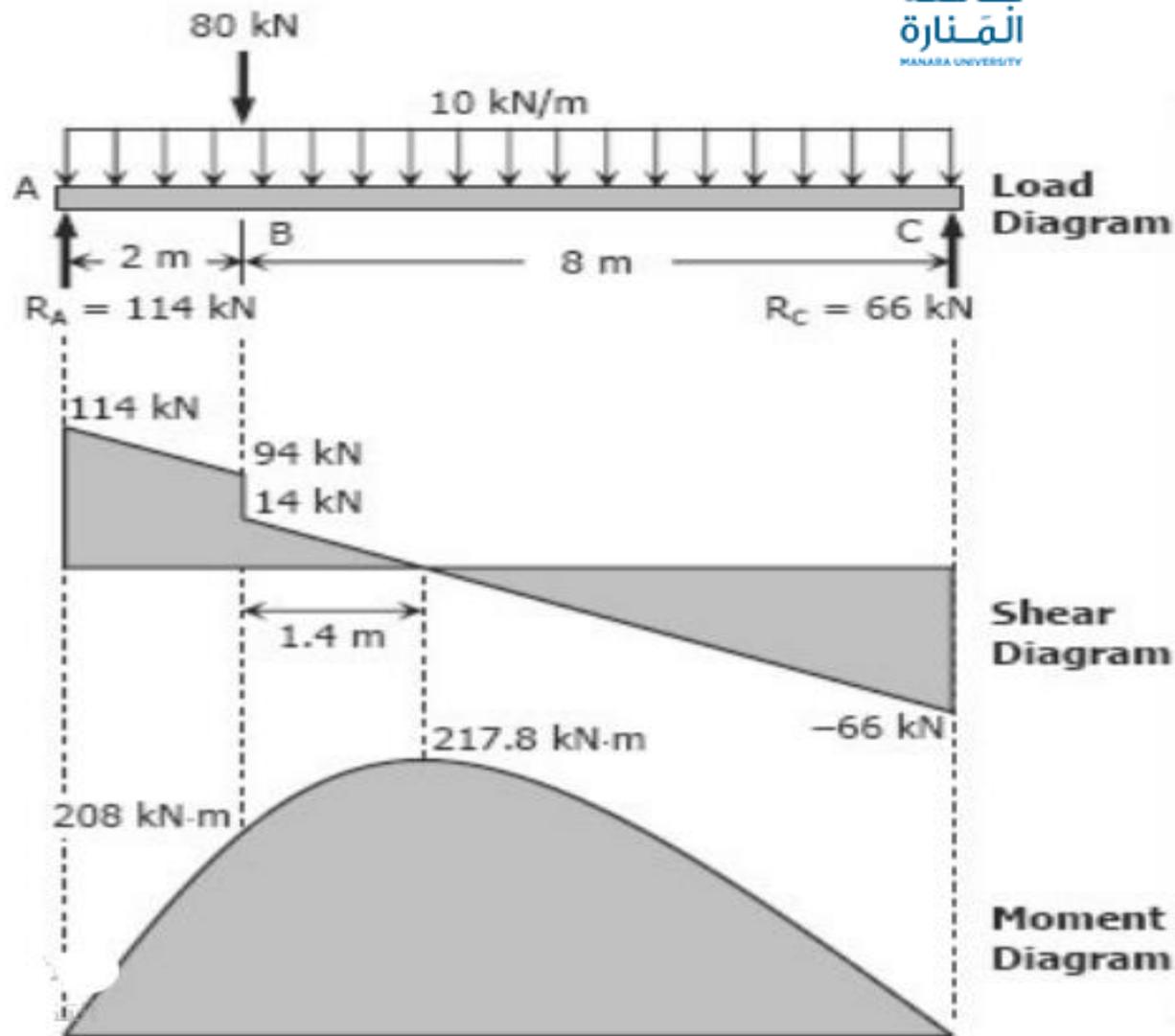
$$M_{AB} = 114x - 10x(x/2) \\ = 114x - 5x^2 \text{ kN}\cdot\text{m}$$

Segment BC:

$$V_{BC} = 114 - 80 - 10x \\ = 34 - 10x \text{ kN}$$

$$M_{BC} = 114x - 80(x - 2) - 10x(x/2) \\ = 160 + 34x - 5x^2$$





من أجل رسم مخطط قوى القص:

To draw the Shear Diagram:

- (1) For segment AB, $V_{AB} = 114 - 10x$ is linear; at $x = 0$, $V_{AB} = 114 \text{ kN}$; at $x = 2 \text{ m}$, $V_{AB} = 94 \text{ kN}$.
- (2) $V_{BC} = 34 - 10x$ for segment BC is linear; at $x = 2 \text{ m}$, $V_{BC} = 14 \text{ kN}$; at $x = 10 \text{ m}$, $V_{BC} = -66 \text{ kN}$. When $V_{BC} = 0$, $x = 3.4 \text{ m}$ thus $V_{BC} = 0$ at 1.4 m from B.

من أجل رسم مخطط عزم الانحناء:

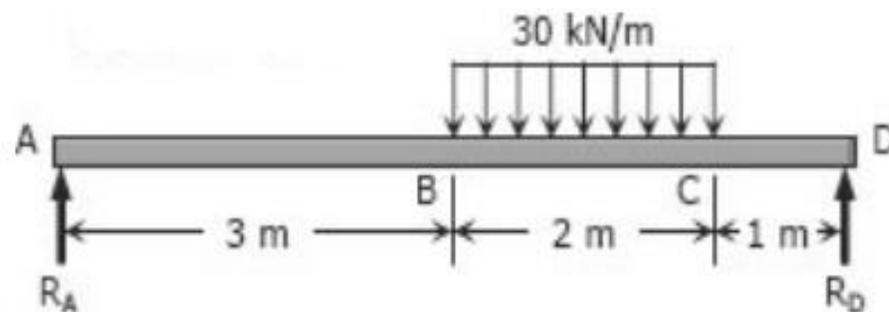
To draw the Moment Diagram:

- (1) $M_{AB} = 114x - 5x^2$ is a second degree curve for segment AB; at $x = 0$, $M_{AB} = 0$; at $x = 2 \text{ m}$, $M_{AB} = 208 \text{ kN-m}$.
- (2) The moment diagram is also a second degree curve for segment BC given by $M_{BC} = 160 + 34x - 5x^2$; at $x = 2 \text{ m}$, $M_{BC} = 208 \text{ kN-m}$; at $x = 10 \text{ m}$, $M_{BC} = 0$.
- (3) Note that the maximum moment occurs at point of zero shear. Thus, at $x = 3.4 \text{ m}$, $M_{BC} = 217.8 \text{ kN-m}$.



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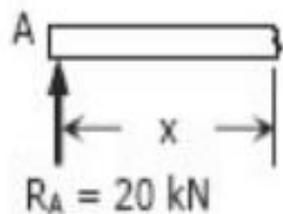
مسألة (3): ارسم مخططات قوى القص وعزوم الانحناء للعتبة المبينة في الشكل.



Segment AB:

$$V_{AB} = 20 \text{ kN}$$

$$M_{AB} = 20x \text{ kN}\cdot\text{m}$$



$$\sum M_A = 0$$

$$6R_D = 4[2(30)]$$

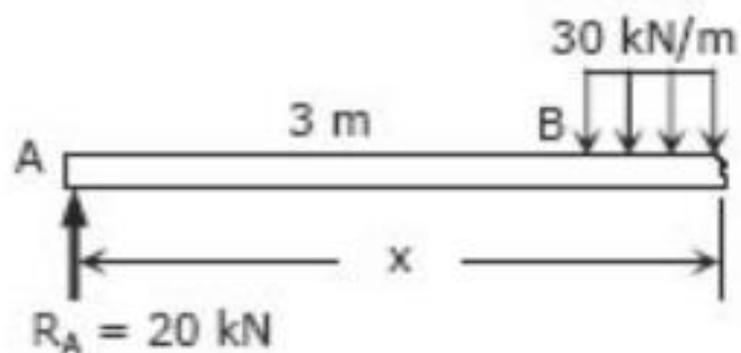
$$R_D = 40 \text{ kN}$$

$$\sum M_D = 0$$

$$6R_A = 2[2(30)]$$

$$R_A = 20 \text{ kN}$$

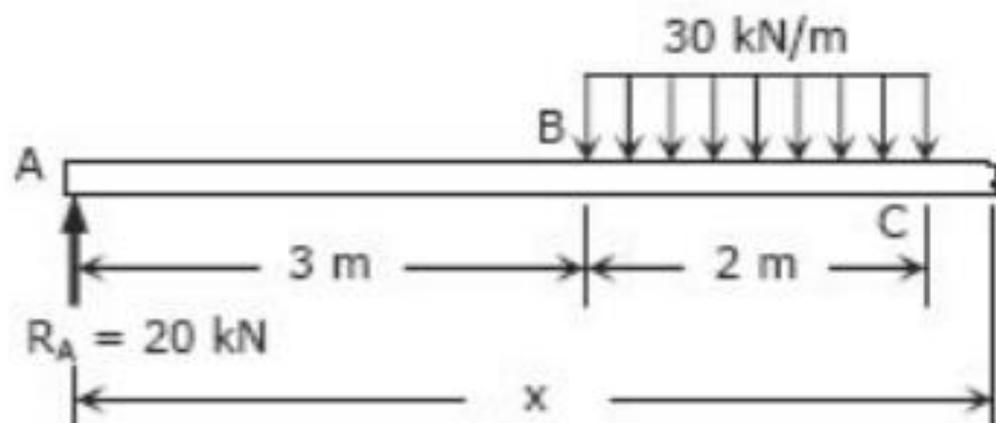
حساب ردود الأفعال



Segment BC:

$$V_{BC} = 20 - 30(x - 3) \\ = 110 - 30x \text{ kN}$$

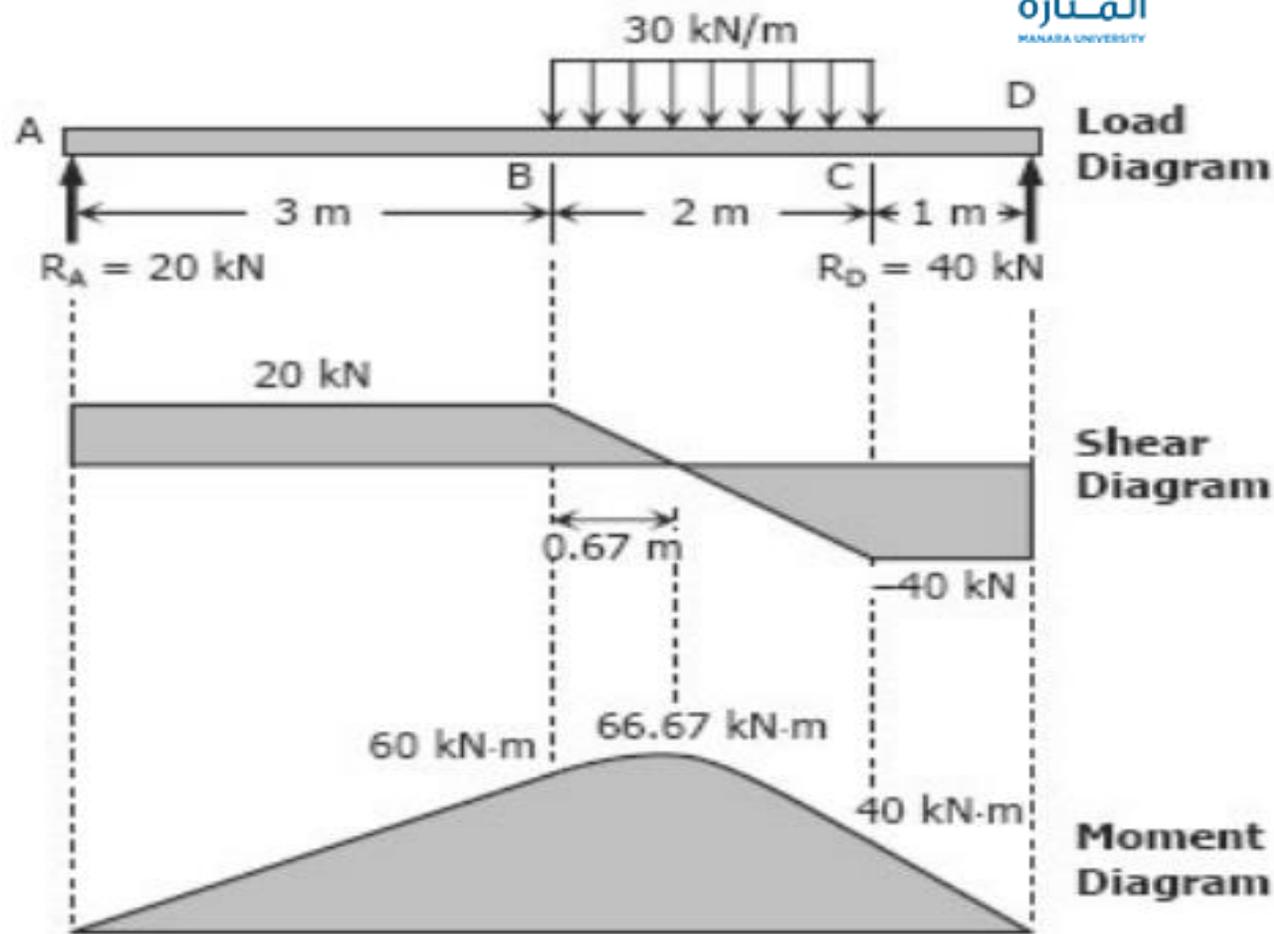
$$M_{BC} = 20x - 30(x - 3)(x - 3)/2 \\ = 20x - 15(x - 3)^2$$



Segment CD:

$$V_{CD} = 20 - 30(2) \\ = -40 \text{ kN}$$

$$M_{CD} = 20x - 30(2)(x - 4) \\ = 20x - 60(x - 4)$$



من أجل رسم مخطط قوى القص:

To draw the Shear Diagram:

- (1) For segment AB, the shear is uniformly distributed at 20 kN.
- (2) $V_{BC} = 110 - 30x$ for segment BC; at $x = 3$ m, $V_{BC} = 20$ kN; at $x = 5$ m, $V_{BC} = -40$ kN. For $V_{BC} = 0$, $x = 3.67$ m or 0.67 m from B.
- (3) The shear for segment CD is uniformly distributed at -40 kN.

من أجل رسم مخطط عزم الانحناء:

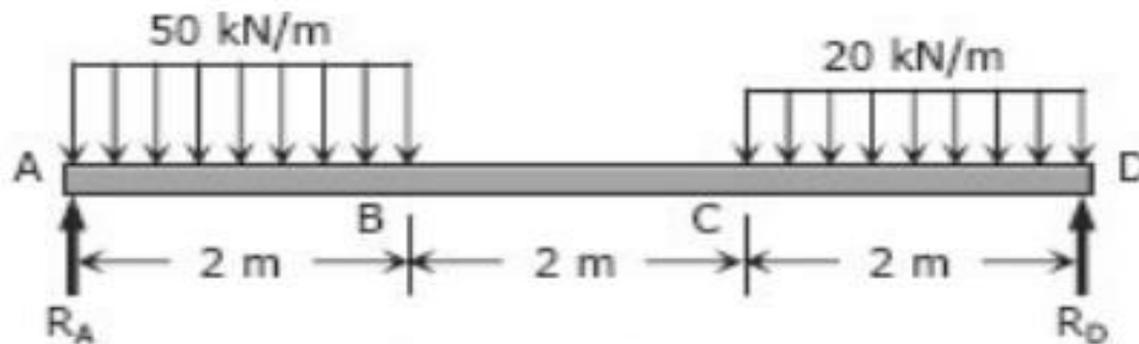
To draw the Moment Diagram:

- (1) For AB, $M_{AB} = 20x$; at $x = 0$, $M_{AB} = 0$; at $x = 3$ m, $M_{AB} = 60$ kN-m.
- (2) $M_{BC} = 20x - 15(x - 3)^2$ for segment BC is second degree curve; at $x = 3$ m, $M_{BC} = 60$ kN-m; at $x = 5$ m, $M_{BC} = 40$ kN-m. Note that maximum moment occurred at zero shear; at $x = 3.67$ m, $M_{BC} = 66.67$ kN-m.
- (3) $M_{CD} = 20x - 60(x - 4)$ for segment BC is linear; at $x = 5$ m, $M_{CD} = 40$ kN-m; at $x = 6$ m, $M_{CD} = 0$.



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مسألة (4) : ارسم مخططات قوى القص وعزوم الانعطاف للعبة المبينة في الشكل.



حساب ردود الأفعال

$$\sum M_A = 0$$

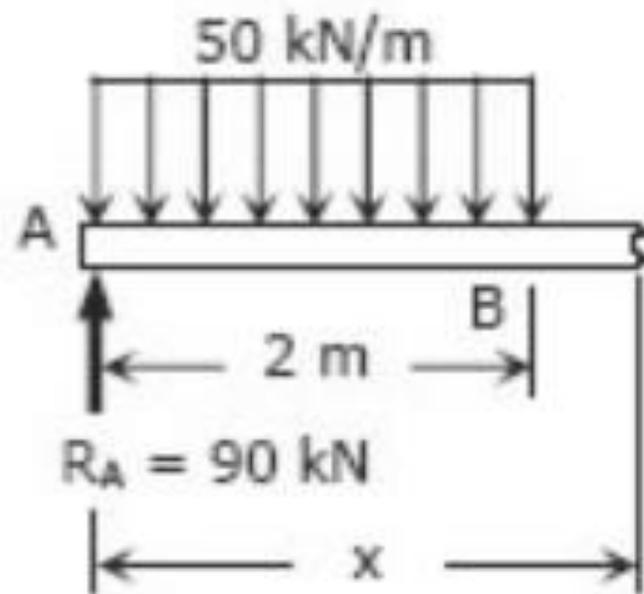
$$6R_D = 1[2(50)] + 5[2(20)]$$

$$R_D = 50 \text{ kN}$$

$$\sum M_D = 0$$

$$6R_A = 5[2(50)] + 1[2(20)]$$

$$R_A = 90 \text{ kN}$$



Segment AB:

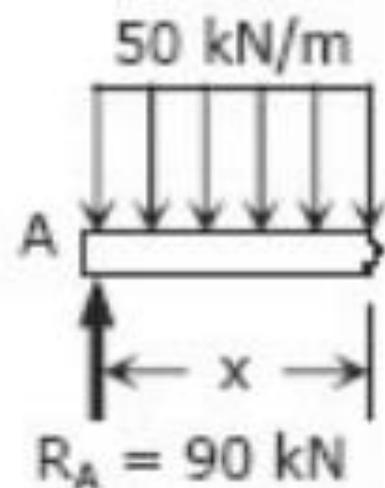
$$V_{AB} = 90 - 50x \text{ kN}$$

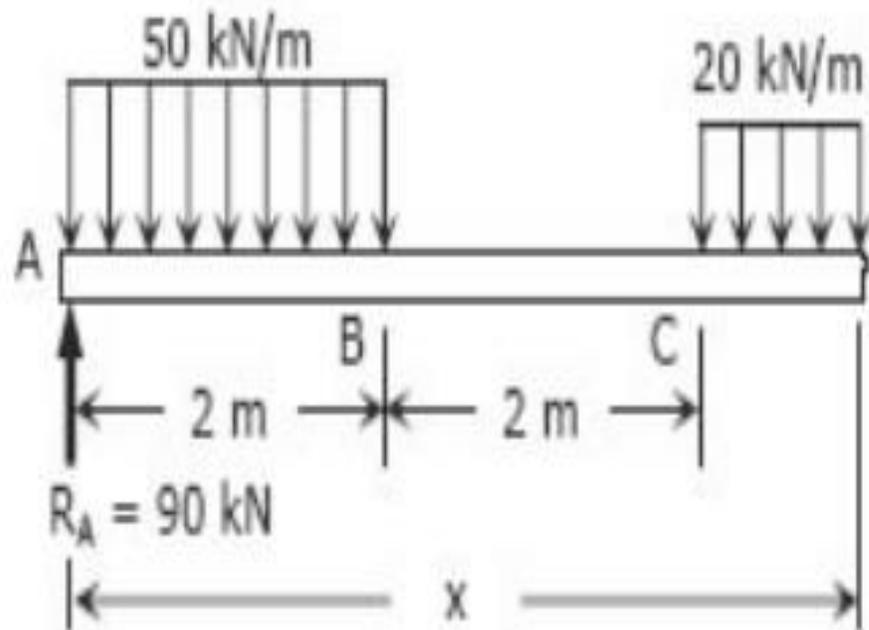
$$M_{AB} = 90x - 50x(x/2) \\ = 90x - 25x^2$$

Segment BC:

$$V_{BC} = 90 - 50(2) \\ = -10 \text{ kN}$$

$$M_{BC} = 90x - 2(50)(x - 1) \\ = -10x + 100 \text{ kN}\cdot\text{m}$$

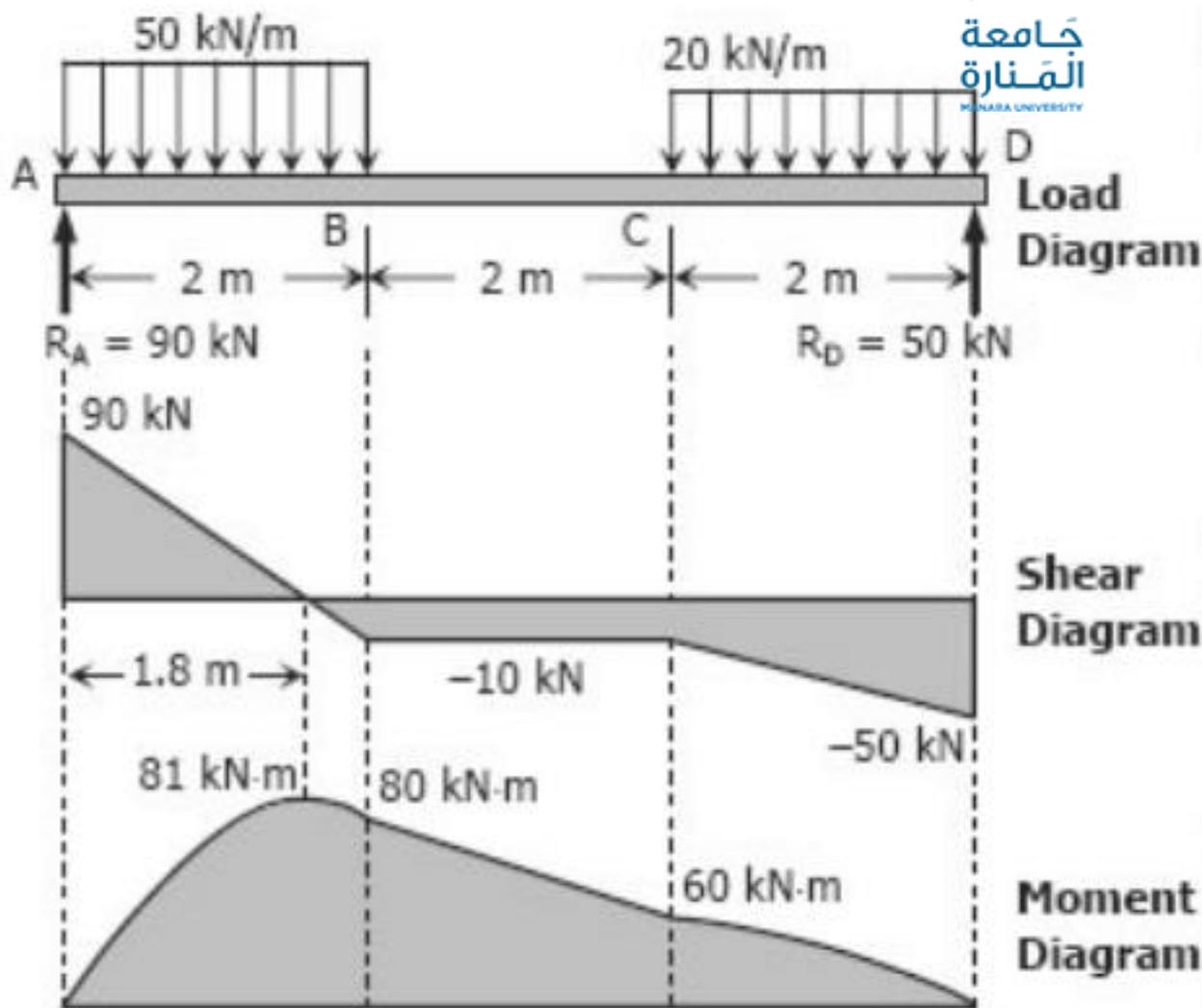




Segment CD:

$$\begin{aligned}V_{CD} &= 90 - 2(50) - 20(x - 4) \\ &= -20x + 70 \text{ kN}\end{aligned}$$

$$\begin{aligned}M_{CD} &= 90x - 2(50)(x - 1) \\ &\quad - 20(x - 4)(x - 4)/2 \\ &= 90x - 100(x - 1) - 10(x - 4)^2 \\ &= -10x^2 + 70x - 60 \text{ kN}\cdot\text{m}\end{aligned}$$



من أجل رسم مخطط قوى القص:

To draw the Shear Diagram:

- (1) $V_{AB} = 90 - 50x$ is linear; at $x = 0$, $V_{BC} = 90$ kN; at $x = 2$ m, $V_{BC} = -10$ kN. When $V_{AB} = 0$, $x = 1.8$ m.
- (2) $V_{BC} = -10$ kN along segment BC.
- (3) $V_{CD} = -20x + 70$ is linear; at $x = 4$ m, $V_{CD} = -10$ kN; at $x = 6$ m, $V_{CD} = -50$ kN.

من أجل رسم مخطط عزم الانحناء:

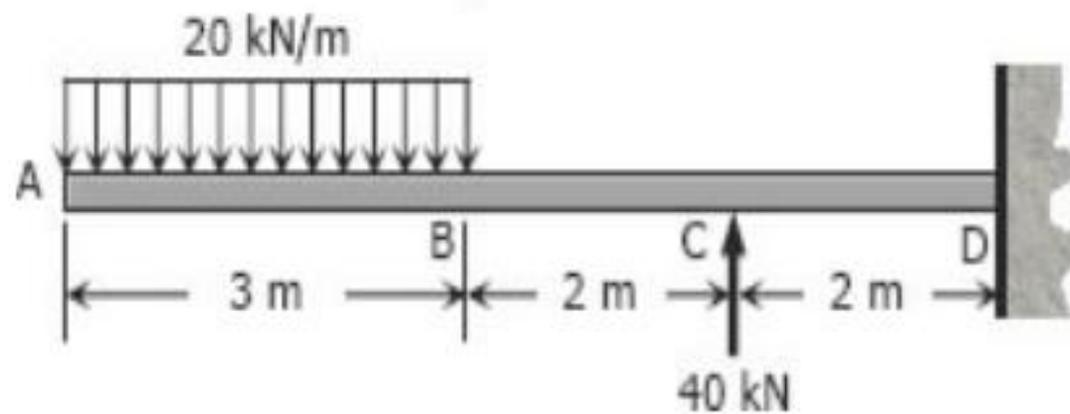
To draw the Moment Diagram:

- (1) $M_{AB} = 90x - 25x^2$ is second degree; at $x = 0$, $M_{AB} = 0$; at $x = 1.8$ m, $M_{AB} = 81$ kN·m; at $x = 2$ m, $M_{AB} = 80$ kN·m.
- (2) $M_{BC} = -10x + 100$ is linear; at $x = 2$ m, $M_{BC} = 80$ kN·m; at $x = 4$ m, $M_{BC} = 60$ kN·m.
- (3) $M_{CD} = -10x^2 + 70x - 60$; at $x = 4$ m, $M_{CD} = 60$ kN·m; at $x = 6$ m, $M_{CD} = 0$.

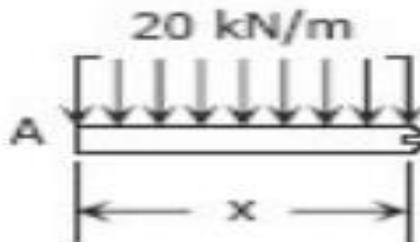


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مسألة (5): ارسم مخططات قوى القص وعزوم الانحناء للعتبة المبينة في الشكل .



Segment AB:
 $V_{AB} = -20x \text{ kN}$
 $M_{AB} = -20x(x/2)$
 $= -10x^2 \text{ kN}\cdot\text{m}$

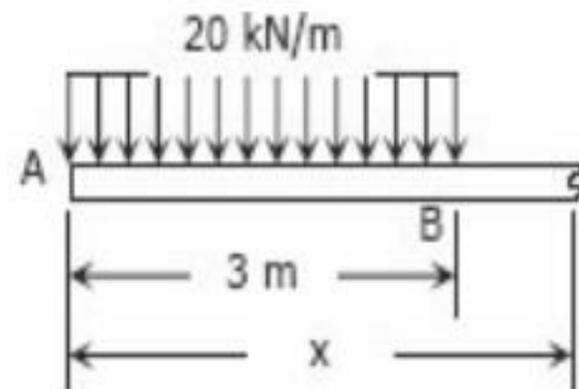
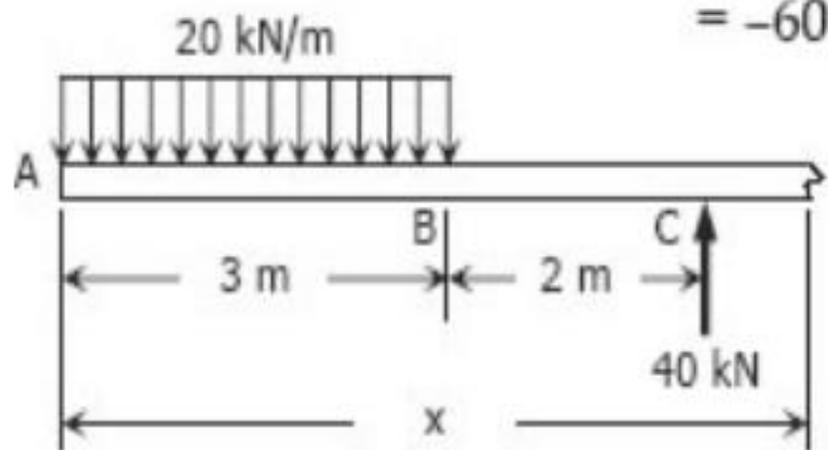




Segment BC:

$$V_{BC} = -20(3) \\ = -60 \text{ kN}$$

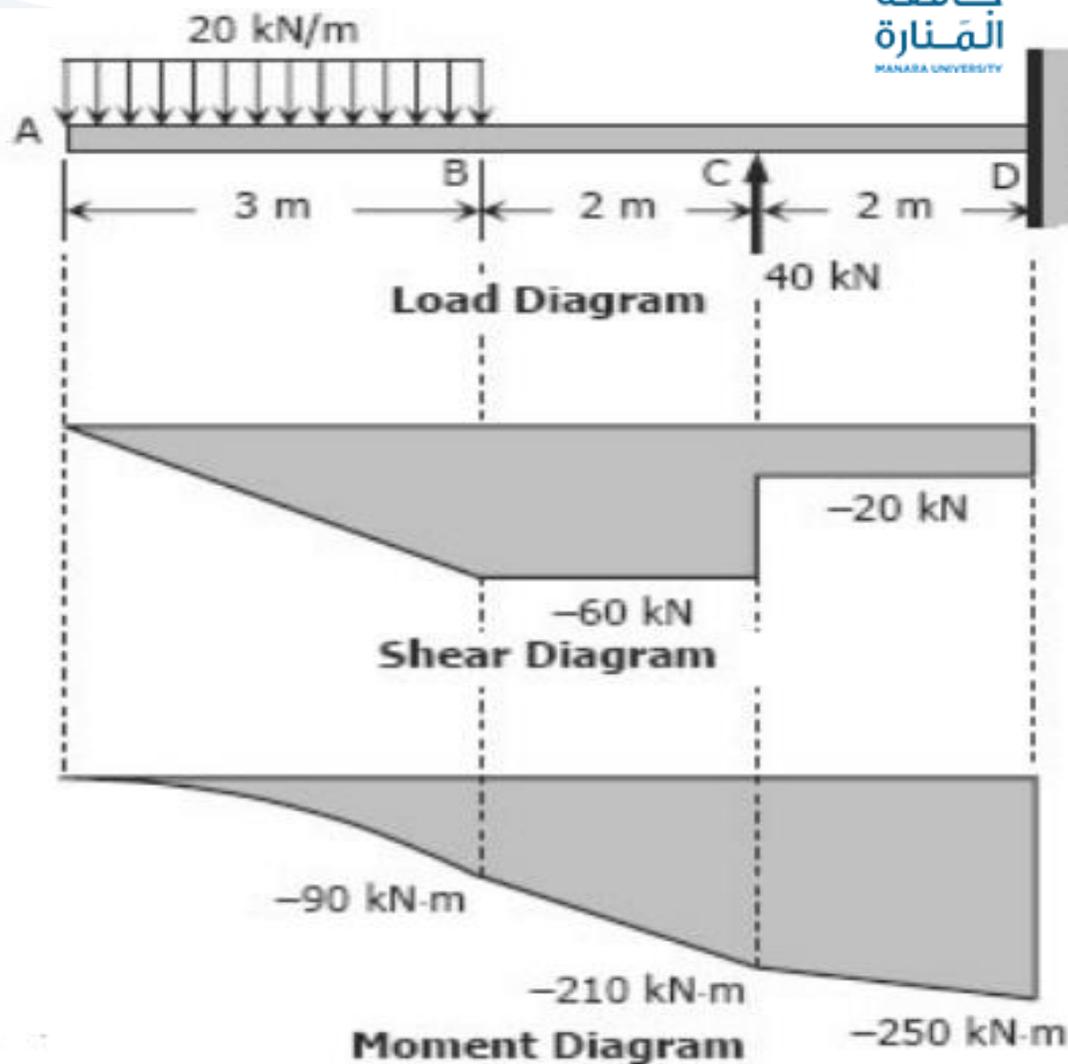
$$M_{BC} = -20(3)(x - 1.5) \\ = -60(x - 1.5) \text{ kN}\cdot\text{m}$$



Segment CD:

$$V_{CD} = -20(3) + 40 \\ = -20 \text{ kN}$$

$$M_{CD} = -20(3)(x - 1.5) + 40(x - 5) \\ = -60(x - 1.5) + 40(x - 5)$$



من أجل رسم مخطط قوى القص:

To draw the Shear Diagram

- (1) $V_{AB} = -20x$ for segment AB is linear; at $x = 0$, $V = 0$; at $x = 3$ m, $V = -60$ kN.
- (2) $V_{BC} = -60$ kN is uniformly distributed along segment BC.
- (3) Shear is uniform along segment CD at -20 kN.

من أجل رسم مخطط عزم الانحناء:

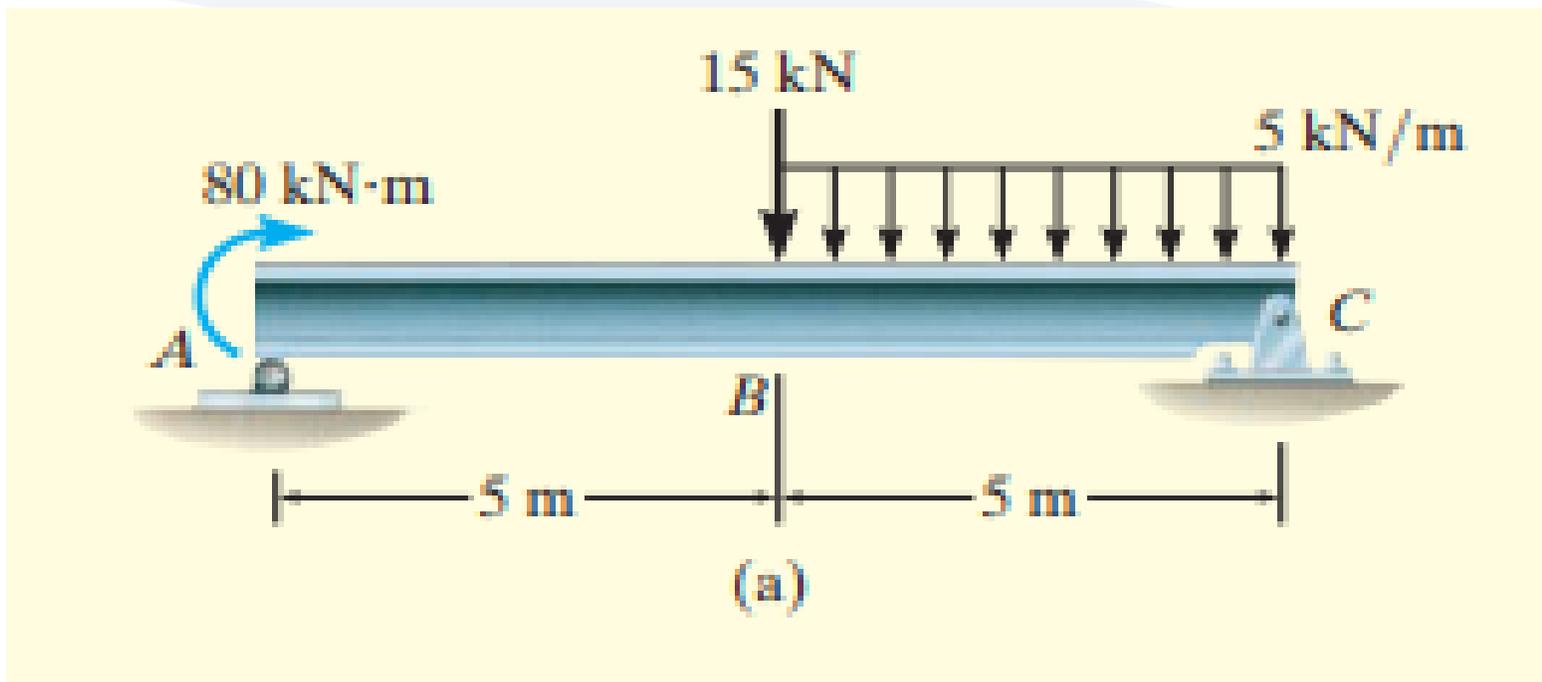
To draw the Moment Diagram

- (1) $M_{AB} = -10x^2$ for segment AB is second degree curve; at $x = 0$, $M_{AB} = 0$; at $x = 3$ m, $M_{AB} = -90$ kN-m.
- (2) $M_{BC} = -60(x - 1.5)$ for segment BC is linear; at $x = 3$ m, $M_{BC} = -90$ kN-m; at $x = 5$ m, $M_{BC} = -210$ kN-m.
- (3) $M_{CD} = -60(x - 1.5) + 40(x - 5)$ for segment CD is also linear; at $x = 5$ m, $M_{CD} = -210$ kN-m, at $x = 7$ m, $M_{CD} = -250$ kN-m.



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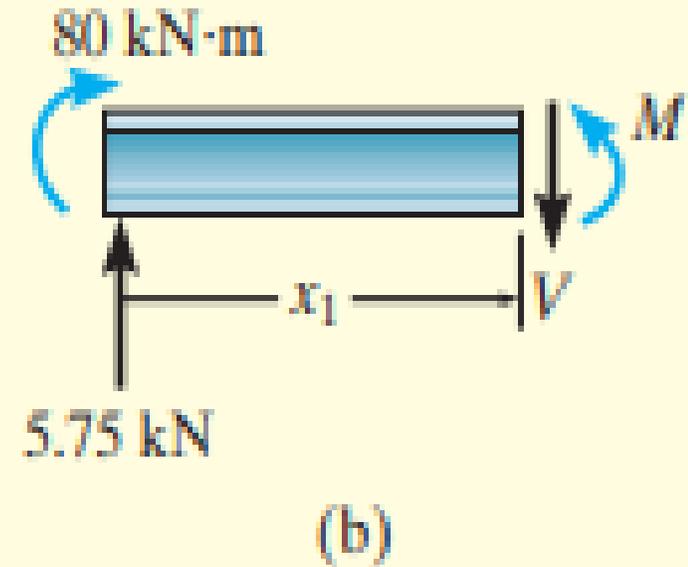
مسألة (6): ارسم مخططات قوى القص وعزوم الانعطاف للعتبة المبينة في الشكل .



$0 \leq x_1 < 5 \text{ m}$, Fig. 6-7b:

$$+\uparrow \sum F_y = 0; \quad 5.75 \text{ kN} - V = 0$$
$$V = 5.75 \text{ kN} \quad (1)$$

$$\zeta + \sum M = 0; \quad -80 \text{ kN}\cdot\text{m} - 5.75 \text{ kN} x_1 + M = 0$$
$$M = (5.75x_1 + 80) \text{ kN}\cdot\text{m} \quad (2)$$



$5 \text{ m} < x_2 \leq 10 \text{ m}$, Fig. 6-7c:

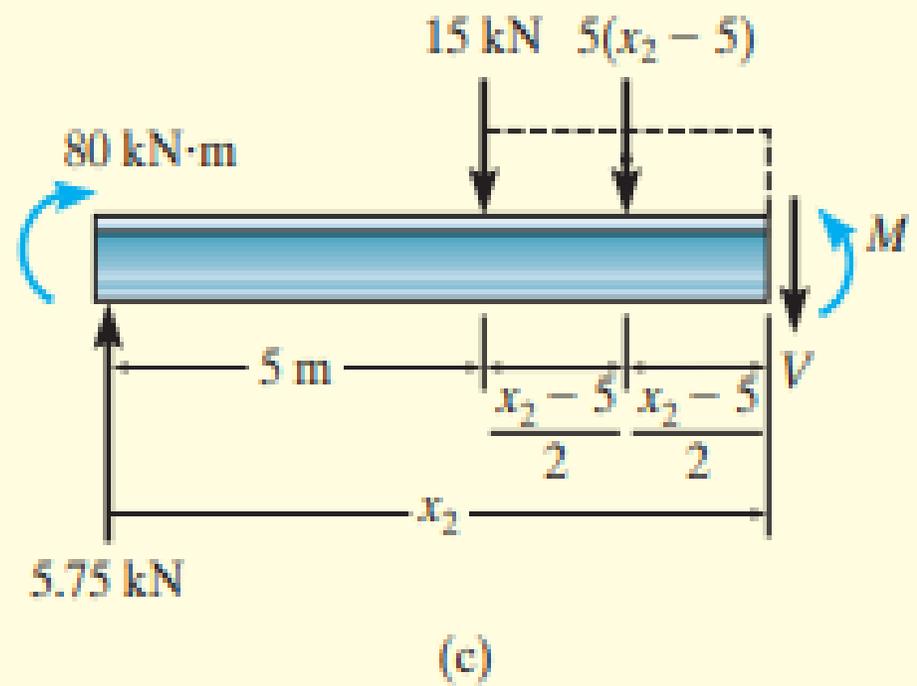
$$+\uparrow \Sigma F_y = 0; \quad 5.75 \text{ kN} - 15 \text{ kN} - 5 \text{ kN/m}(x_2 - 5 \text{ m}) - V = 0$$

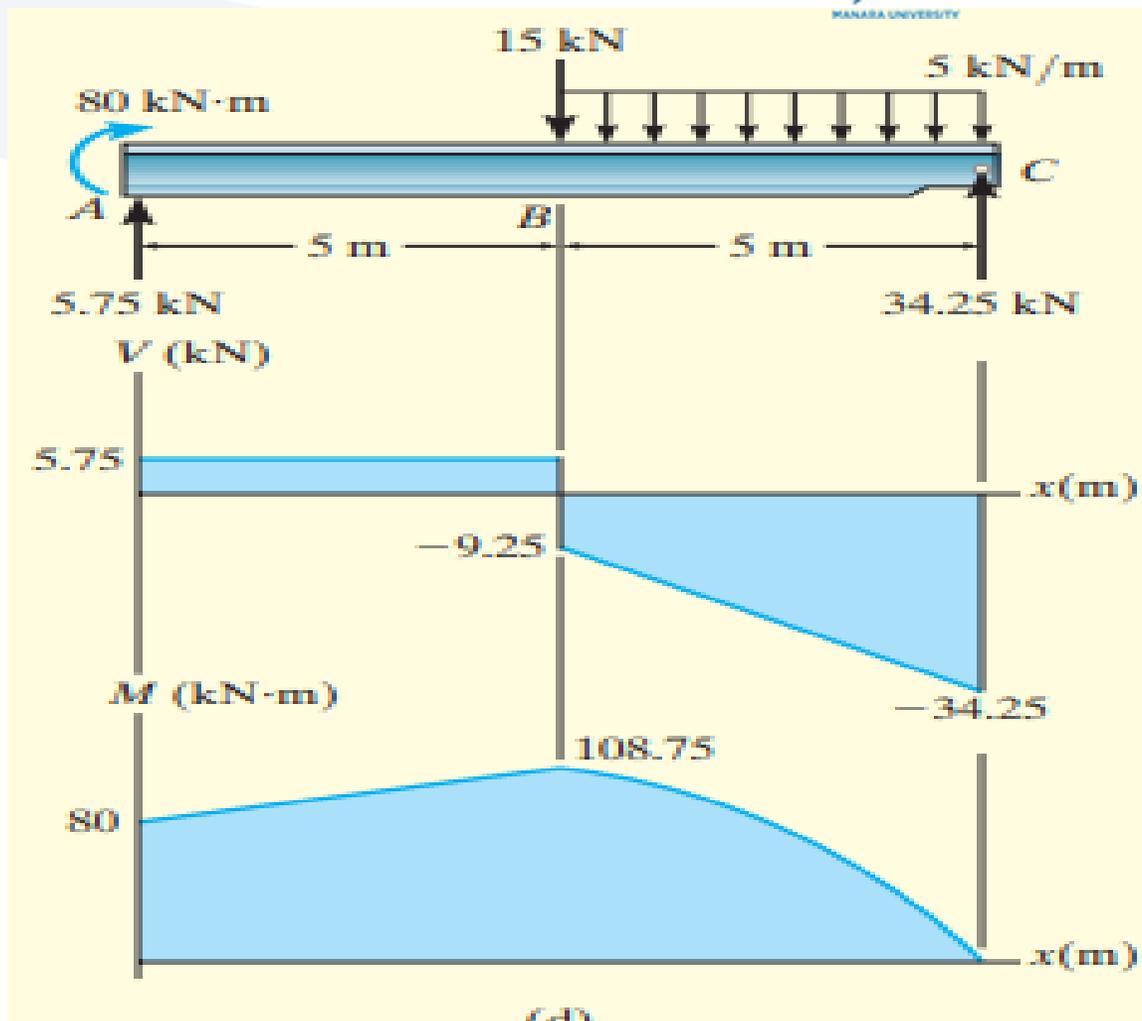
$$V = (15.75 - 5x_2) \text{ kN} \quad (3)$$

$$\downarrow + \Sigma M = 0; \quad -80 \text{ kN}\cdot\text{m} - 5.75 \text{ kN} x_2 + 15 \text{ kN}(x_2 - 5 \text{ m})$$

$$+ 5 \text{ kN/m}(x_2 - 5 \text{ m}) \left(\frac{x_2 - 5 \text{ m}}{2} \right) + M = 0$$

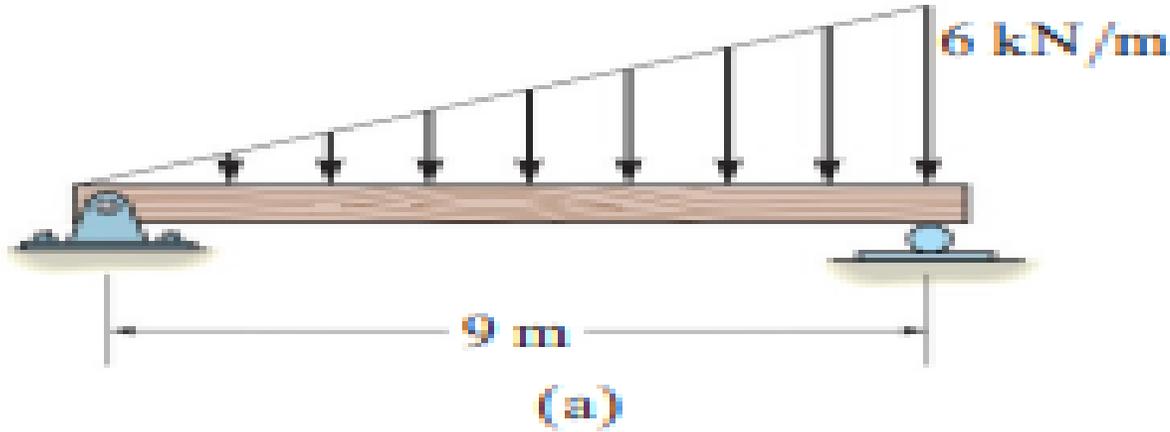
$$M = (-2.5x_2^2 + 15.75x_2 + 92.5) \text{ kN}\cdot\text{m} \quad (4)$$





رسم مخطط قوى القص وعزم الانحناء:

مسألة (7): ارسم مخططات قوى القص وعزوم الانعطاف للعتبة المبينة في الشكل .

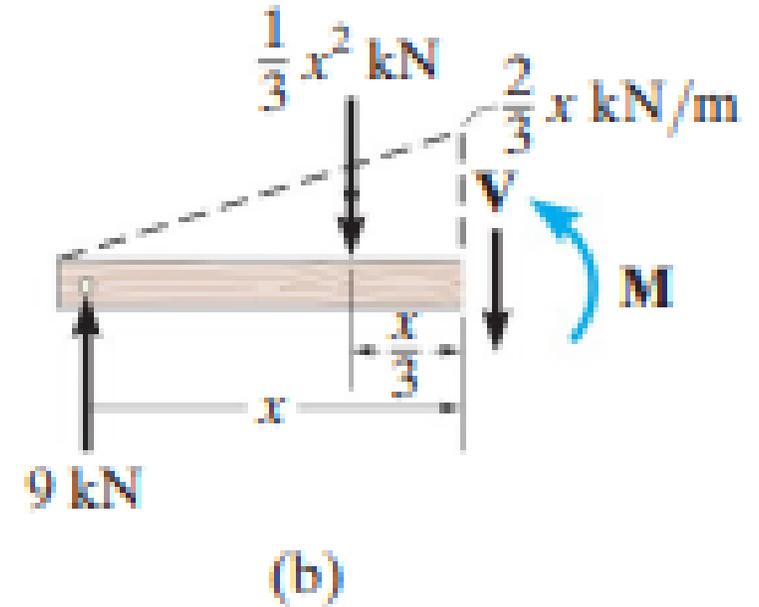


$$+\uparrow \Sigma F_y = 0; \quad 9 - \frac{1}{3}x^2 - V = 0$$

$$V = \left(9 - \frac{x^2}{3}\right) \text{ kN} \quad (1)$$

$$\zeta + \Sigma M = 0; \quad M + \frac{1}{3}x^2 \left(\frac{x}{3}\right) - 9x = 0$$

$$M = \left(9x - \frac{x^3}{9}\right) \text{ kN} \cdot \text{m} \quad (2)$$





Shear and Moment Diagrams. The shear and moment diagrams shown in Fig. 7-12c are obtained by plotting Eqs. 1 and 2.

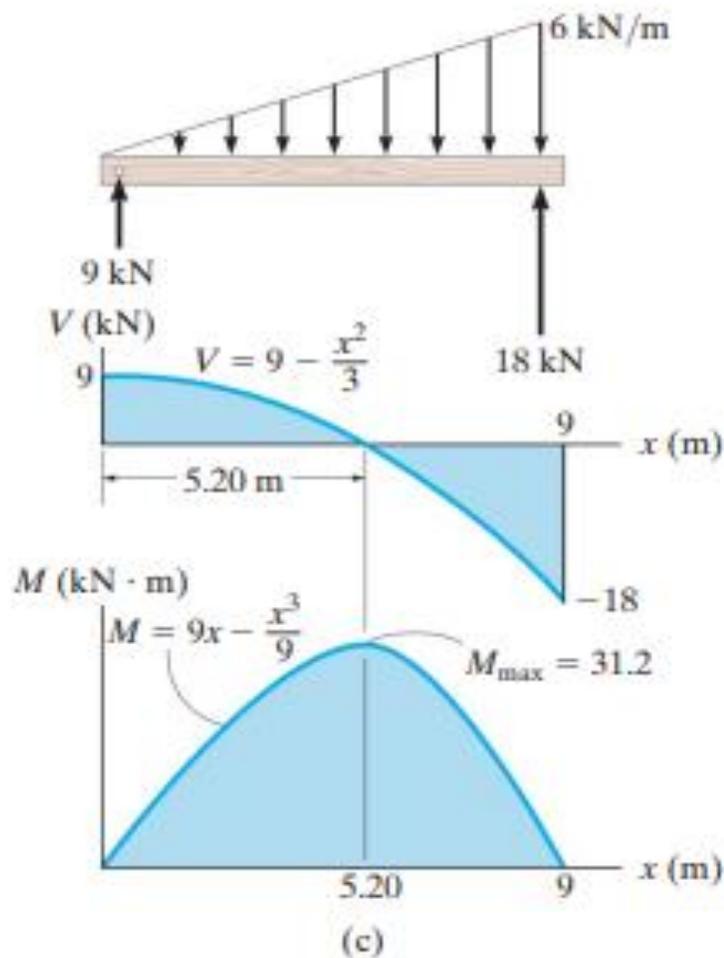
The point of *zero shear* can be found using Eq. 1:

$$V = 9 - \frac{x^2}{3} = 0$$

$$x = 5.20 \text{ m}$$

NOTE: It will be shown in Sec. 7.3 that this value of x happens to represent the point on the beam where the *maximum moment* occurs. Using Eq. 2, we have

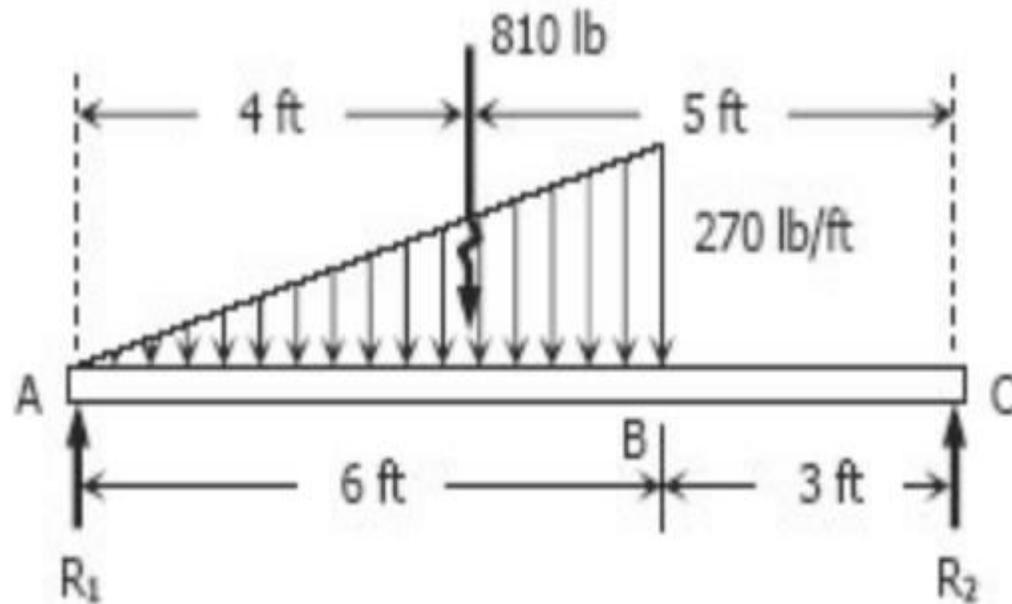
$$\begin{aligned} M_{\max} &= \left(9(5.20) - \frac{(5.20)^3}{9} \right) \text{ kN} \cdot \text{m} \\ &= 31.2 \text{ kN} \cdot \text{m} \end{aligned}$$





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مسألة (8): ارسم مخططات قوى القص وعزوم الانحناء للعتبة المبينة في الشكل.



$$[\sum M_C = 0]$$

$$9R_1 = 5(810)$$

$$R_1 = 450 \text{ lb}$$

$$[\sum M_A = 0]$$

$$9R_2 = 4(810)$$

$$R_2 = 360 \text{ lb}$$



Segment AB:

$$\frac{y}{x} = \frac{270}{6}$$

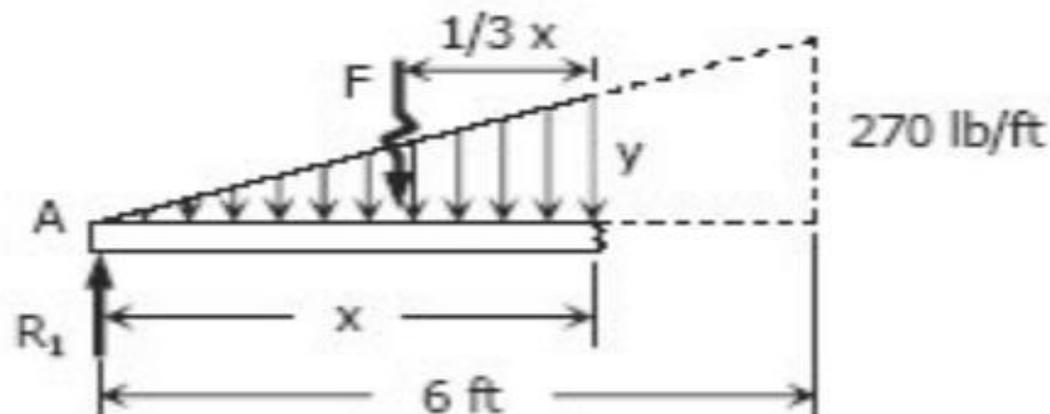
$$y = 45x$$

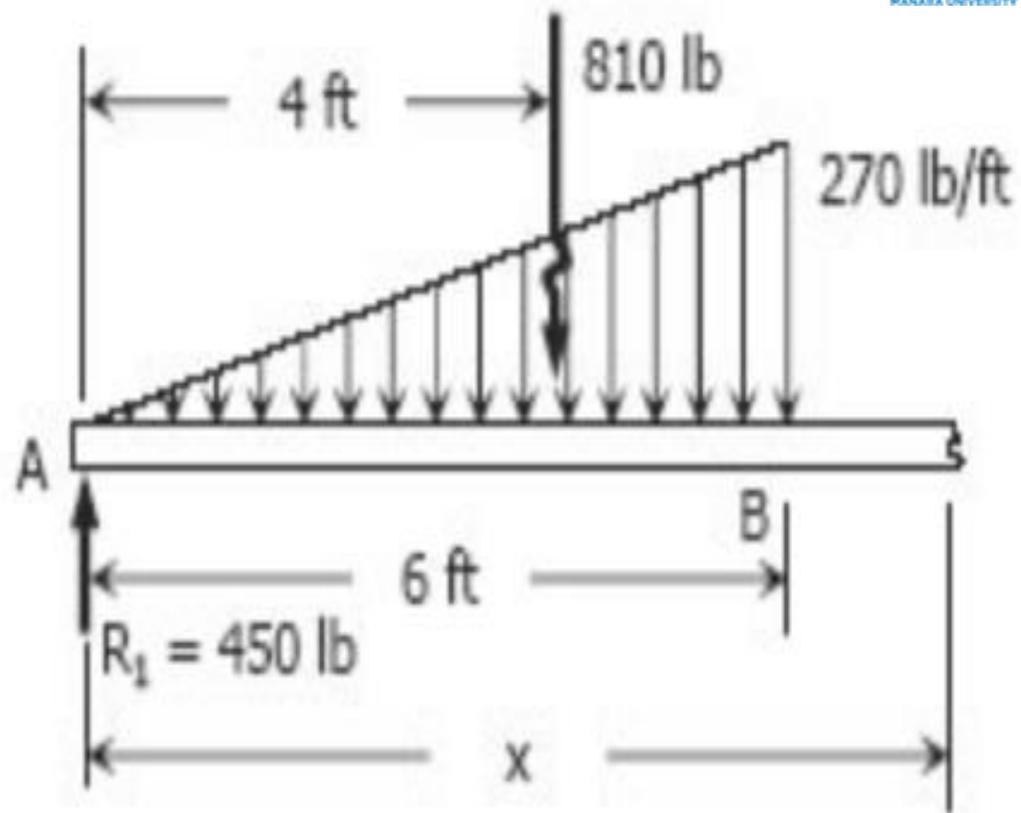
$$F = \frac{1}{2}xy = \frac{1}{2}x(45x)$$

$$F = 22.5x^2$$

$$\begin{aligned} V_{AB} &= R_1 - F \\ &= 450 - 22.5x^2 \text{ lb} \end{aligned}$$

$$\begin{aligned} M_{AB} &= R_1x - F\left(\frac{1}{3}x\right) \\ &= 450x - 22.5x^2\left(\frac{1}{3}x\right) \\ &= 450x - 7.5x^3 \text{ lb-ft} \end{aligned}$$





Segment BC:

$$\begin{aligned} V_{BC} &= 450 - 810 \\ &= -360 \text{ lb} \end{aligned}$$

$$\begin{aligned} M_{BC} &= 450x - 810(x - 4) \\ &= 450x - 810x + 3240 \\ &= 3240 - 360x \text{ lb}\cdot\text{ft} \end{aligned}$$



من أجل رسم مخطط قوى القص:

To draw the Shear Diagram:

- (1) $V_{AB} = 450 - 22.5x^2$ is a second degree curve; at $x = 0$, $V_{AB} = 450$ lb; at $x = 6$ ft, $V_{AB} = -360$ lb.
- (2) At $x = a$, $V_{AB} = 0$,
 $450 - 22.5x^2 = 0$
 $22.5x^2 = 450$
 $x^2 = 20$
 $x = \sqrt{20}$

To check, use the squared property of parabola.

$$a^2/450 = 6^2/(450 + 360)$$

$$a^2 = 20$$

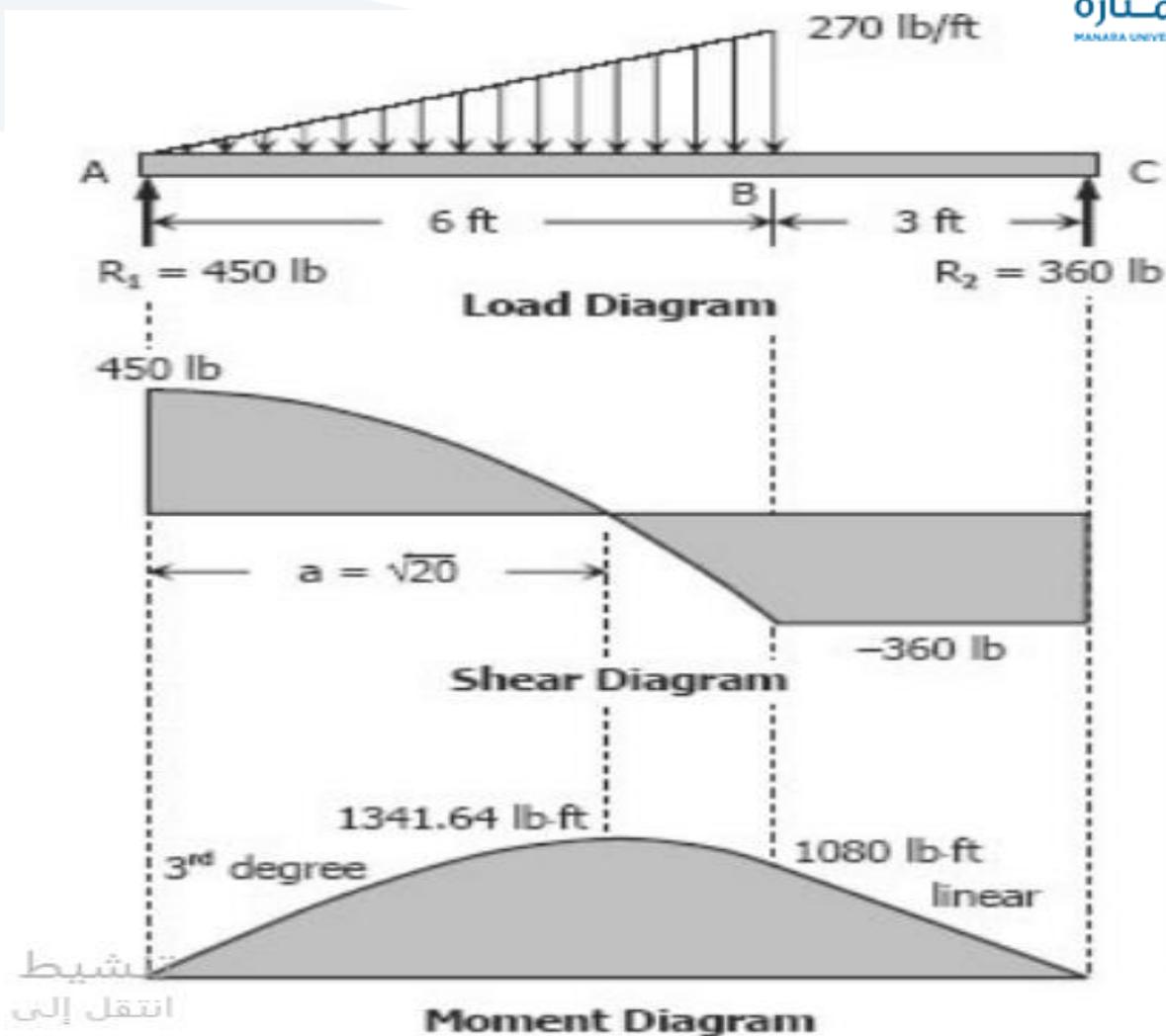
$$a = \sqrt{20}$$

- (3) $V_{BC} = -360$ lb is constant.

من أجل رسم مخطط عزم الانحناء:

To draw the Moment Diagram:

- (1) $M_{AB} = 450x - 7.5x^3$ for segment AB is third degree curve; at $x = 0$, $M_{AB} = 0$; at $x = \sqrt{20}$, $M_{AB} = 1341.64$ lb-ft; at $x = 6$ ft, $M_{AB} = 1080$ lb-ft.
- (2) $M_{BC} = 3240 - 360x$ for segment BC is linear; at $x = 6$ ft, $M_{BC} = 1080$ lb-ft; at $x = 9$ ft, $M_{BC} = 0$.



تشيط
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