



# Introduction to Artificial Intelligence

## Problem Solving and Search

### Part I

#### Lecture 3

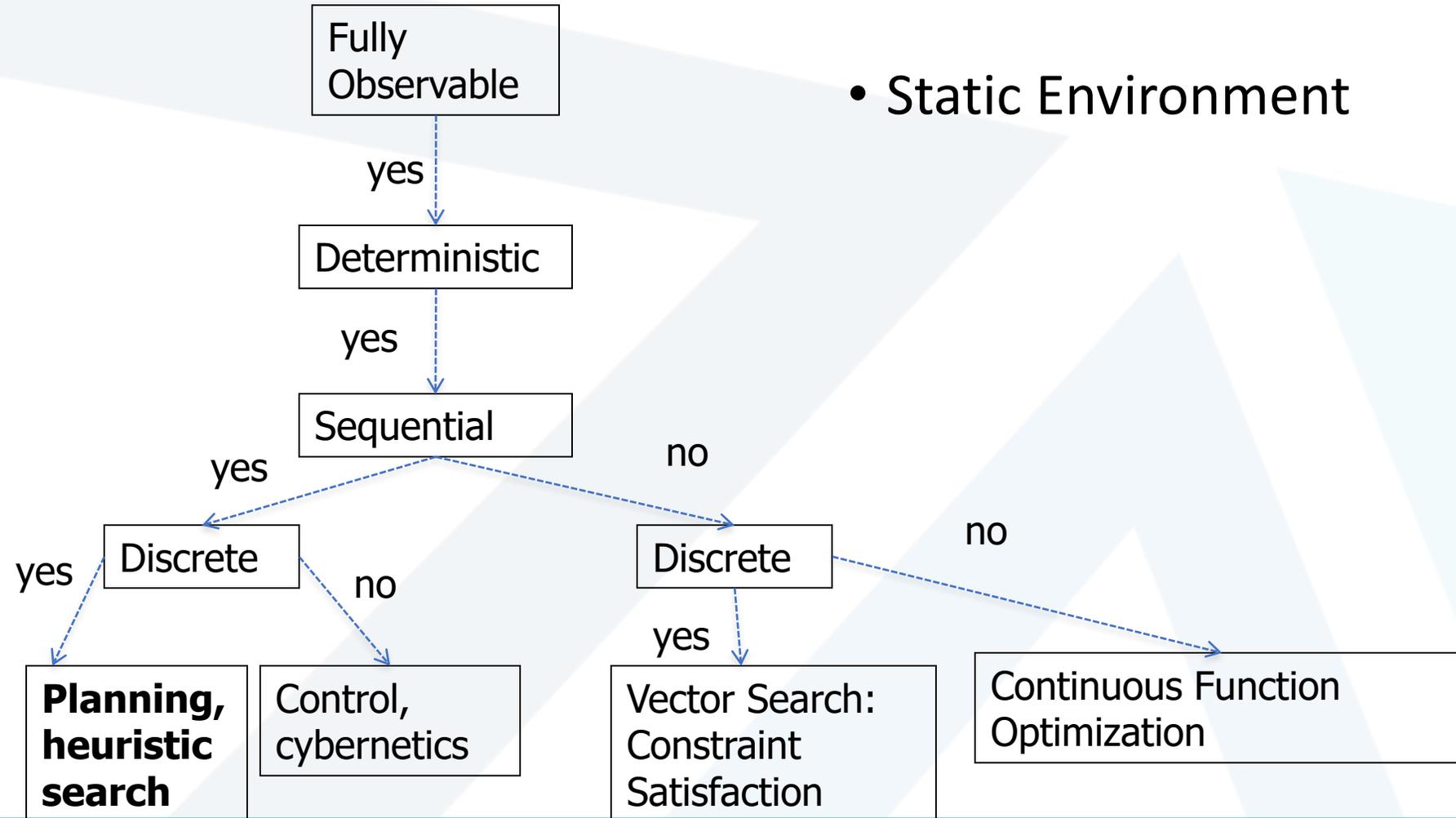
Prof Dr. Eng. Mariam M. Saii

# Outline

- Problem formulation: representing sequential problems.
- Example problems.
- Planning for solving sequential problems without uncertainty.
- Basic search algorithms
  - BFS
  - DFS
  - iterative

# Environment Type Discussed In this Lecture

- Static Environment



## الاستراتيجية العامة لحل المسائل

- التمثيل العام لحل المسائل
- تطبيق عمليات للانتقال من حالة الى اخرى
- السعي للوصول الى الهدف

## مسألة المبشرين وأكلة لحوم البشر

- ثلاث مبشرين وثلاث من أكلة لحوم البشر على الضفة نهر ( الضفة اليسرى ) ويريدون الانتقال الى الضفة الأخرى (الضفة اليمنى للنهر )
- يوجد قارب يتسع لشخصين فقط
- يجب الا يزيد عدد أكلة لحوم البشر على عدد المبشرين في أي مرحلة من مراحل الانتقال

## مسألة المبشرين وأكلة لحوم البشر

- \* تدل على أن الحالة غير جائزة وهي زيادة عدد أكلة لحوم البشر على عدد المبشرين

LEFT BANK الضفة اليسرى			RIGHT BANK الضفة اليمنى		
M	C	BOAT	M	C	BOAT
0	0	YES	3	3	NO
0	1	YES	3	2	NO
0	2	YES	3	1	NO
0	3	YES	3	0	NO
1	0	YES	2	3*	NO
1	1	YES	2	2	NO
1	2*	YES	2	1	NO
1	3*	YES	2	0	NO
2	0	YES	1	3*	NO
2	1	YES	1	2*	NO
2	2	YES	1	1	NO
2	3*	YES	1	0	NO
3	0	YES	0	3	NO
3	1	YES	0	2	NO
3	2	YES	0	1	NO
3	3	YES	0	0	NO
0	0	NO	3	3	YES
0	1	NO	3	2	YES
0	2	NO	3	1	YES
0	3	NO	3	0	YES
1	0	NO	2	3*	YES
1	1	NO	2	2	YES
1	2*	NO	2	1	YES
1	3*	NO	2	0	YES
2	0	NO	1	3*	YES
2	1	NO	1	2*	YES
2	2	NO	1	1	YES
2	3*	NO	1	0	YES
3	0	NO	0	3	YES
3	1	NO	0	2	YES
3	2	NO	0	1	YES
3	3	NO	0	0	YES

## مسألة المبشرين وأكلة لحوم البشر

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0	1	YES	3	2	NO
0	2	YES	3	1	NO
0	3	YES	3	0	NO
1	1	YES	2	2	NO
2	2	YES	1	1	NO
3	0	YES*	0	3	NO
3	1	YES	0	2	NO
3	2	YES	0	1	NO
3	3	YES	0	0	NO
0	0	NO	3	3	YES
0	1	NO	3	2	YES
0	2	NO	3	1	YES
0	3	NO*	3	0	YES
1	1	NO	2	2	YES
2	2	NO	1	1	YES
3	0	NO	0	3	YES
3	1	NO	0	2	YES
3	2	NO	0	1	YES
3	3	NO*	0	0	YES

- \* تدل على أن الحالة تتنافى مع الفرض (غير جائزة) وهي زيادة عدد أكلة لحوم البشر على عدد المبشرين

## مسألة المبشرين وأكلة لحوم البشر – العمليات الممكنة

العمليات الممكنة	العملية
انقل مبشراً من الضفة اليسرى إلى اليمنى	01
انقل متوحشاً من الضفة اليسرى إلى اليمنى	02
انقل مبشراً ومتوحشاً من الضفة اليسرى إلى اليمنى	03
انقل مبشرين من الضفة اليسرى إلى اليمنى	04
انقل متوحشين من الضفة اليسرى إلى اليمنى	05
انقل مبشراً من الضفة اليمنى إلى اليسرى	06
انقل متوحشاً من الضفة اليمنى إلى اليسرى	07
انقل مبشراً ومتوحشاً من الضفة اليمنى إلى اليسرى	08
انقل مبشرين من الضفة اليمنى إلى اليسرى	09
انقل متوحشين من الضفة اليمنى إلى اليسرى	010



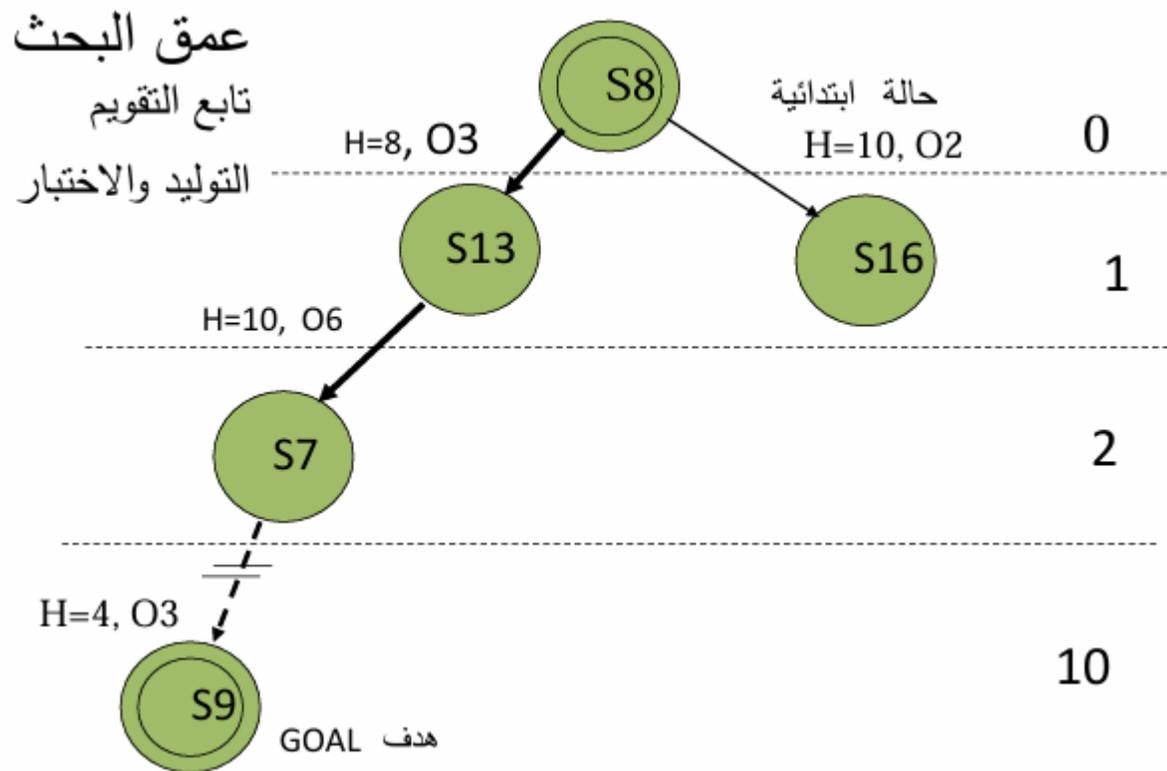
## مسألة المبشرين وأكلة لحوم البشر- الحالات الممكنة

الحالة الموجودة	الضفة اليسرى LEFT BANK			الضفة اليمنى RIGHT BANK		
	M	C	BOAT	M	C	BOAT
S1	0	1	YES	3	2	NO
S2	0	2	YES	3	1	NO
S3	0	3	YES	3	0	NO
S4	1	1	YES	2	2	NO
S5	2	2	YES	1	1	NO
S6	3	1	YES	0	2	NO
S7	3	2	YES	0	1	NO
S8	3	3	YES	0	0	NO
S9	0	0	NO	3	3	YES
S10	0	1	NO	3	2	YES
S11	0	2	NO	3	1	YES
S12	1	1	NO	2	2	YES
S13	2	2	NO	1	1	YES
S14	3	0	NO	0	3	YES
S15	3	1	NO	0	2	YES
S16	3	2	NO	0	1	YES

## مسألة المبشرين وأكلة لحوم البشر - الانتقالات

الحالة الموجودة		الضفة اليسرى LEFT BANK			الضفة اليمنى RIGHT BANK		
		M	C	BOAT	M	C	BOAT
O3	S8	3	3	YES	0	0	NO
O6	S12	2	2	NO	1	1	YES
O5	S7	3	2	YES	0	1	NO
O7	S6	3	0	NO	0	3	YES
O4	S12	3	1	YES	0	2	NO
O8	S5	1	1	NO	2	2	YES
O4	S11	2	2	YES	1	1	NO
O7	S3	0	2	NO	3	1	YES
O5	S10	0	3	YES	3	0	NO
O3	S4	0	1	NO	3	2	YES
G	S9	0	0	NO	3	3	YES

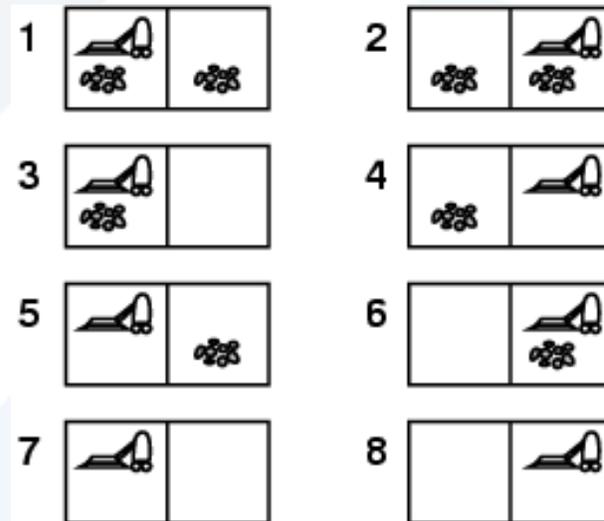
## مسألة المبشرين وأكلة لحوم البشر – مخطط الحالة



# Sequential Action Example

- **Deterministic, fully observable** → **single-state problem**
  - Agent knows exactly which state it will be in; solution is a sequence
  - Vacuum world → everything observed
  - Romania → The full map is observed

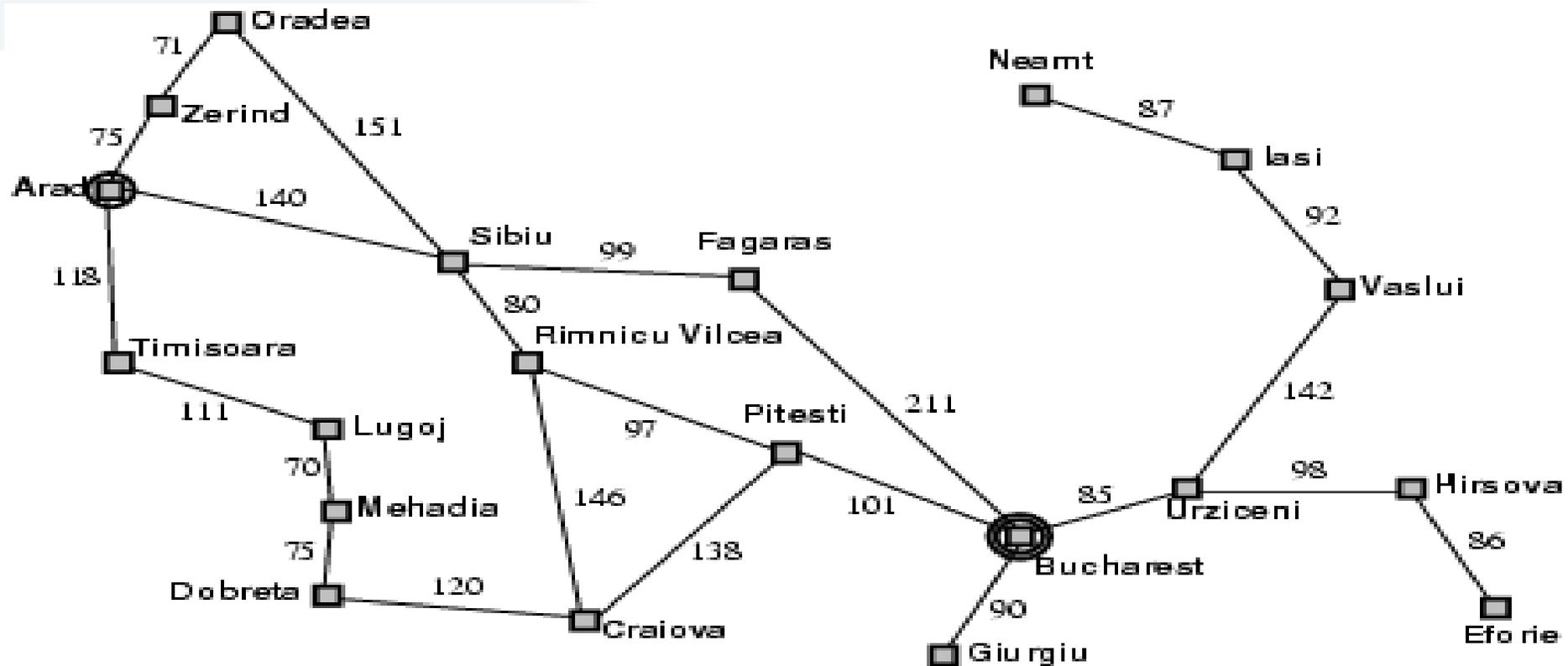
- Single-state: Start in #5. Solution??
  - [Right, Suck]



# Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- 
- **Formulate goal:**
  - be in Bucharest
  -
- **Formulate problem:**
  - **states:** various cities
  - **actions:** drive between cities
- **Find solution:**
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest
  -

# Example: Romania



Abstraction: The process of removing details from a representation  
 Is the map a good representation of the problem? What is a good replacement?



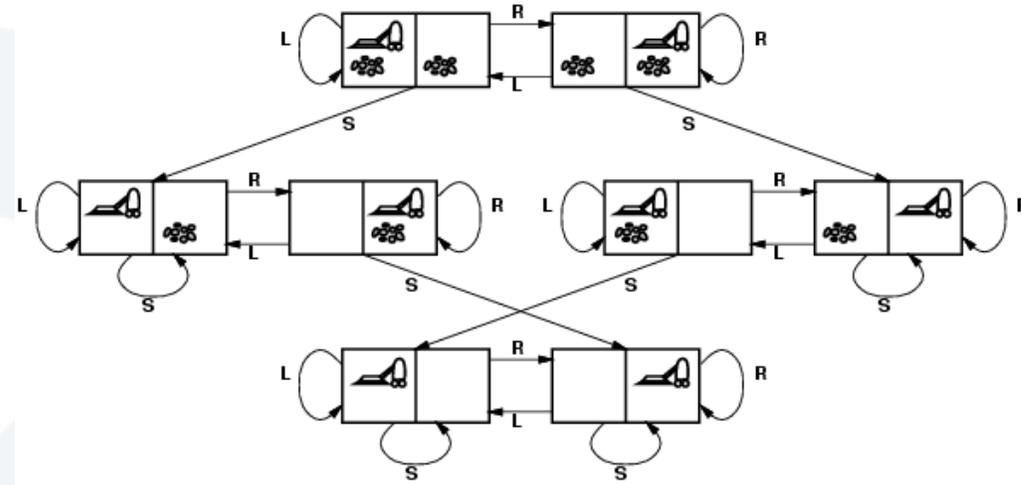
# Single-state problem formulation

A **problem** is defined by four items:

1. **initial state** e.g., "at Arad"
  2. **actions** or **successor function**  $S(x)$  = set of action–state pairs
    - e.g.,  $S(\text{Arad}) = \{ \langle \text{Arad} \rightarrow \text{Zerind}, \text{Zerind} \rangle, \dots \}$
  3. **goal test**, can be
    - **explicit**, e.g.,  $x = \text{"at Bucharest"}$
    - **implicit**, e.g.,  $\text{Checkmate}(x)$
  4. **path cost** (additive)
    - e.g., sum of distances, number of actions executed, etc.
    - $c(x, a, y)$  is the **step cost**, assumed to be  $\geq 0$
- A **solution** is
    - a sequence of actions leading from the initial state to a goal state



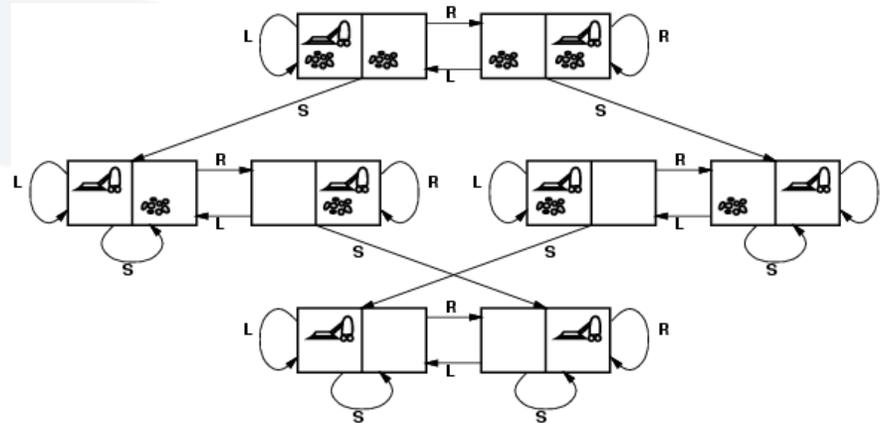
# Vacuum world state space graph



- states?
- actions?
- goal test?
- path cost?
-



# Vacuum world state space graph



- states? integer dirt and robot location
- actions? *Left, Right, Suck*
- goal test? no dirt at all locations
- path cost? 1 per action



# Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- states?
- actions?
- goal test?
- path cost?



# Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

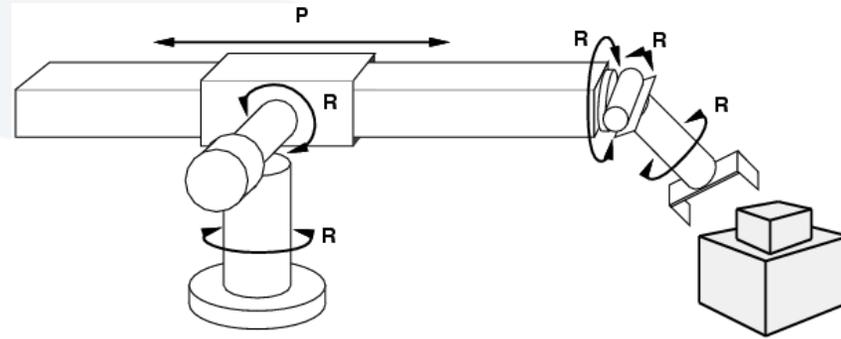
Goal State

- states? locations of tiles
- actions? move blank left, right, up, down
- goal test? = goal state (given)
- path cost? 1 per move

[Note: optimal solution of  $n$ -Puzzle family is NP-hard]

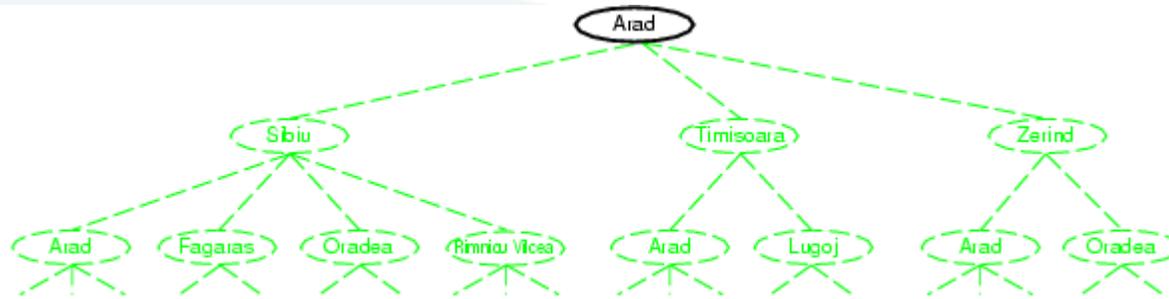


# Example: robotic assembly

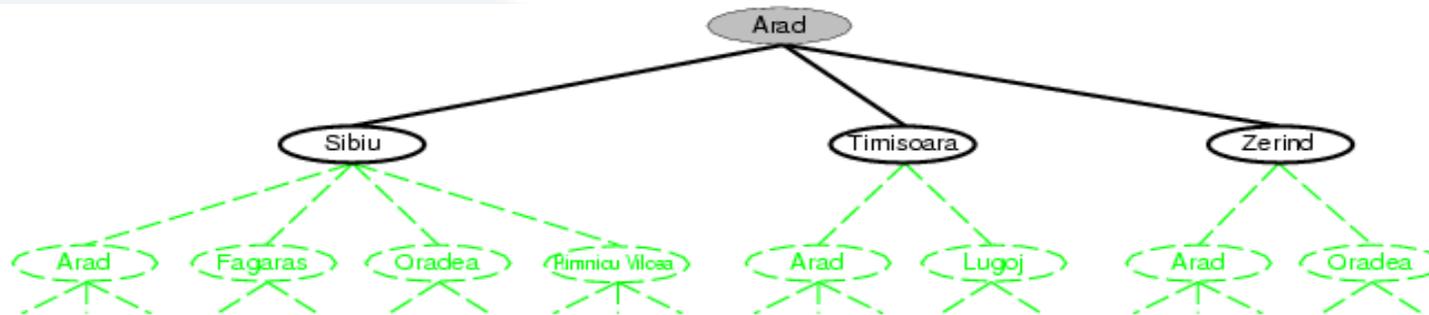


- states?:
  - real-valued coordinates of robot joint angles parts of the object to be assembled
  -
- actions?:
  - continuous motions of robot joints
- goal test?:
  - complete assembly
  -
- path cost?:
  - time to execute
  -

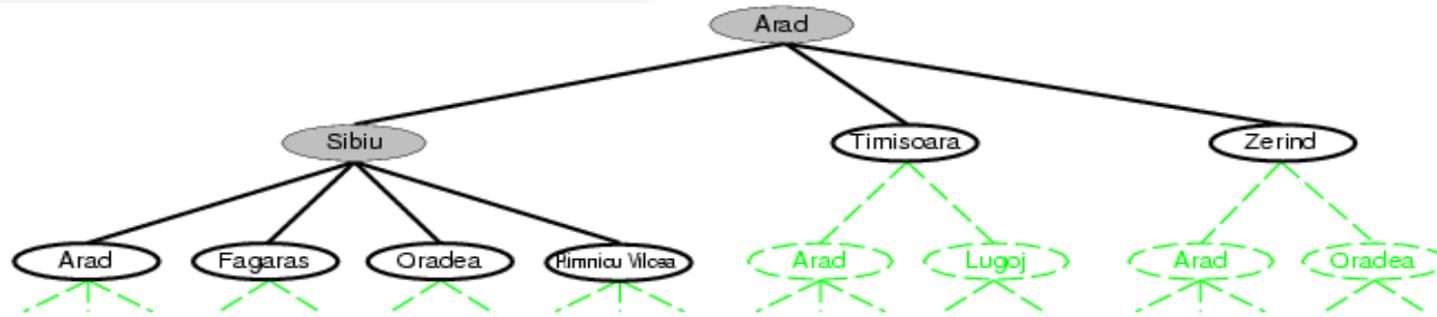
# Tree search example



# Tree search example



# Tree search example



# Search Graph vs. State Graph

- Be careful to distinguish
  - Search tree: nodes are **sequences of actions**.
  - State Graph: Nodes are states of the environment.
  - We will also consider soon **search graphs**.
- Demo: <http://aispace.org/search/>

# معايير تقويم البحث Search strategies

- Strategies are evaluated along the following dimensions:
  - **completeness الشمولية**: does it always find a solution if one exists?
    - تعبر عما اذا كانت الاستراتيجية تضمن الوصول الى الحل في حال وجوده
  - **time complexity**: number of nodes generated
    - كم نستغرق للوصول الى الحل ويرتبط بعدد العقد الموسعة
  - **space complexity**: maximum number of nodes in memory
    - يعبر عن حجم الذاكرة المستهلكة في الحالة الأسوأ ويرتبط بالعقد المخزنة في الذاكرة
  - **optimality**: does it always find a least-cost solution?
    - هل نصل دائما الى الحل ذو الكلفة الأقل ( العمق والممر الأفضل)

# Time and space complexity

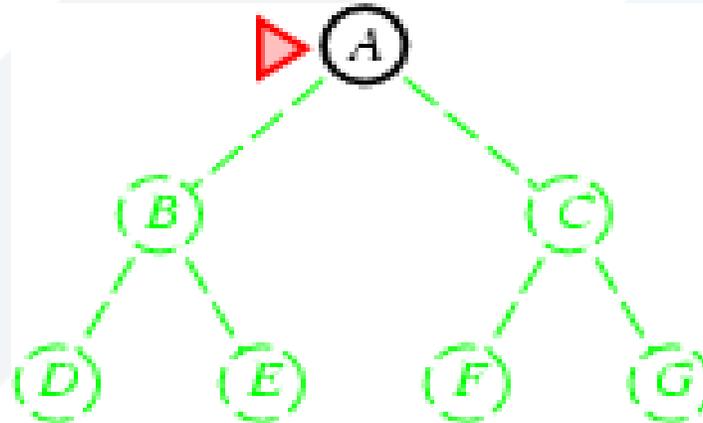
- Time and space complexity are measured in terms of
  - $b$ : maximum branching factor of the search **معامل التفرع الأعظمي**  
لشجرة البحث (عدد العقد الأبناء الأعظمي التي تم توليدها)
  - $d$ : depth of the least-cost solution **عمق الحل الأقل تكلفة**
  - $m$ : maximum depth of the state space (may be  $\infty$ ) **العمق الأعظمي**  
لفضاء الحالة ( قد يكون لا نهائي )

# Search strategies

- **Uninformed** البحث الأعمى search strategies use only the information available in the problem definition
  - Breadth-first search
  - Depth-first search
  - Depth-limited search
  - Iterative deepening search
- **informed** البحث المعرفي search strategies use knowledge about the problem definition
  - Best- first search
  - A\* search

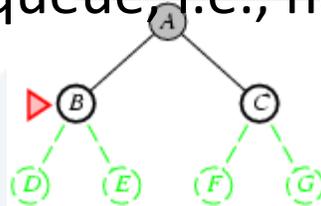
# Breadth-first search

- Expand shallowest unexpanded node
- 
- **Implementation:**
  - *Frontier* or *fringe* is a FIFO queue, i.e., new successors go at end
  -



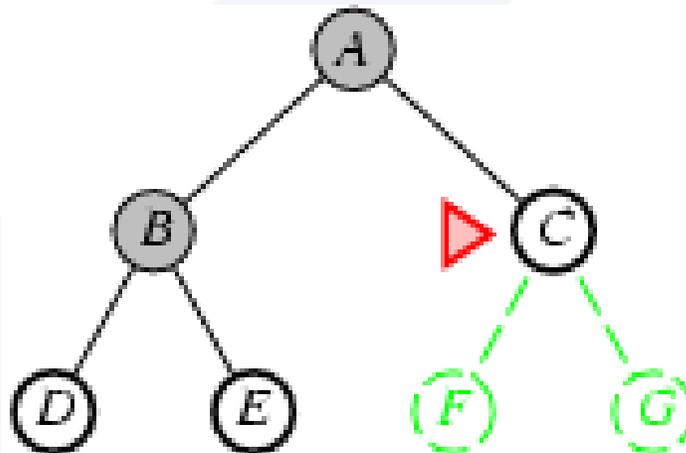
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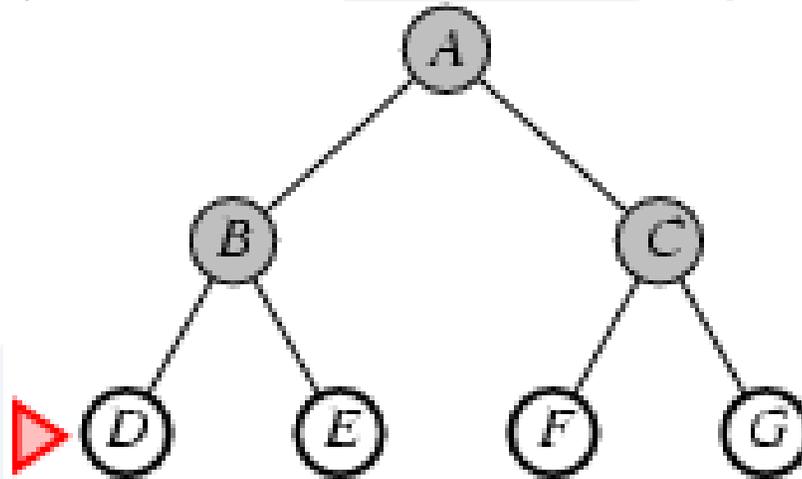
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# Breadth-first search

- Expand shallowest unexpanded node
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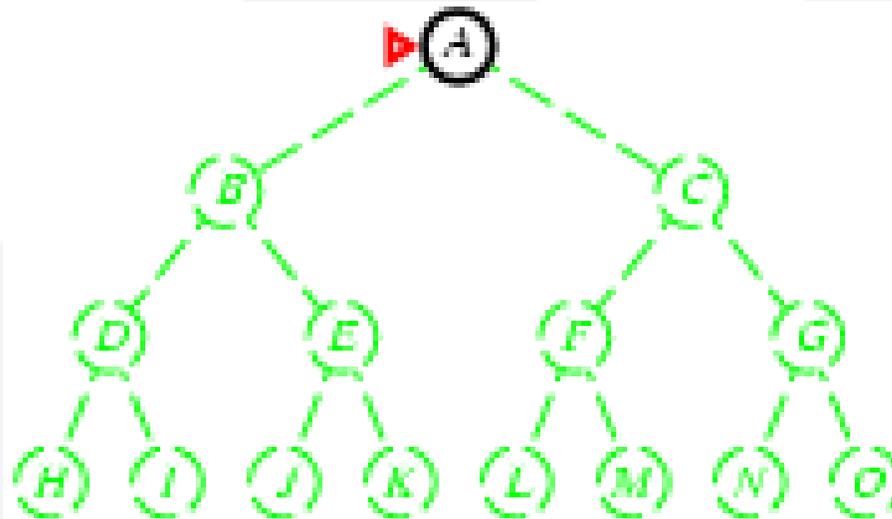


# Properties of breadth-first search

- Complete? Time? Space? Optimal?
- Complete? Yes (if  $b$  is finite)
- 
- Time?  $1+b+b^2+b^3+\dots +b^d + b(b^d-1) = O(b^{d+1})$
- 
- Space?  $O(b^{d+1})$  (keeps every node in memory)
- 
- Optimal? Yes (if cost = 1 per step)
- 
- **Space** is the bigger problem (more than time)
-

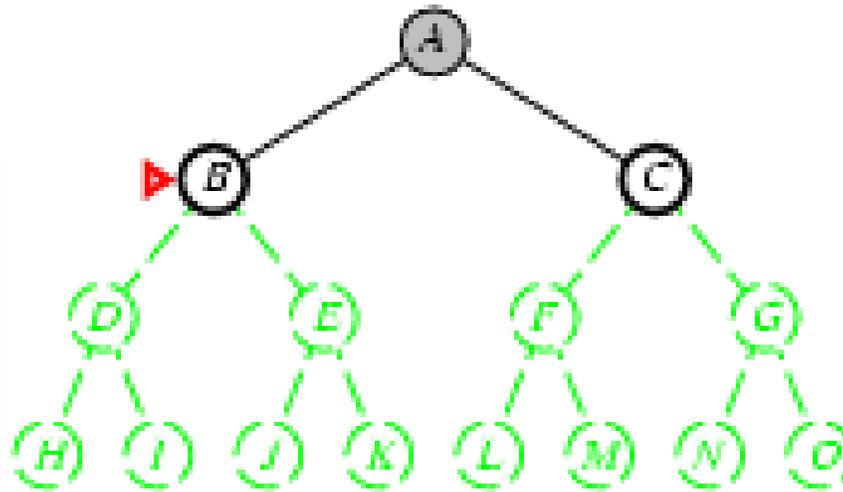
# Depth-first search

- Expand deepest unexpanded node
- 
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front
  -



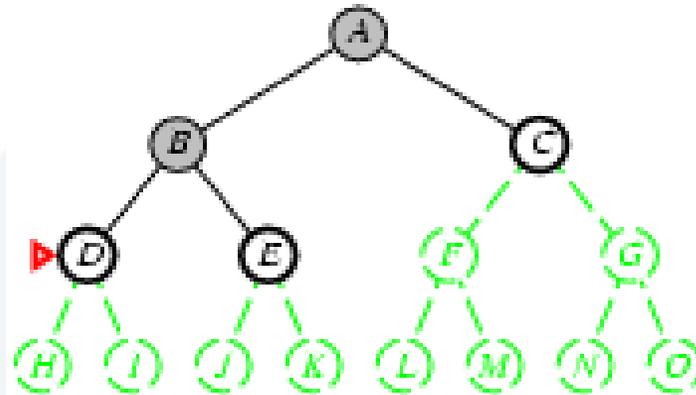
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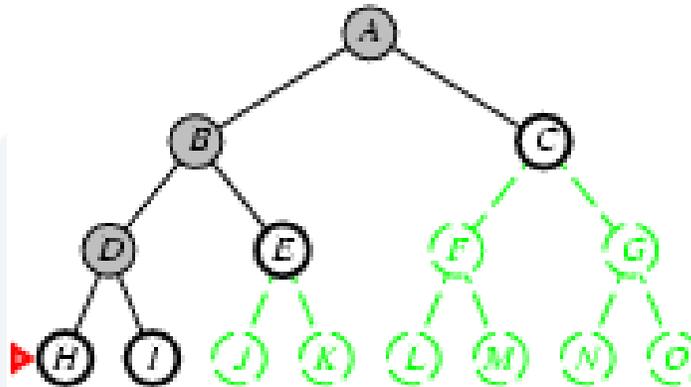
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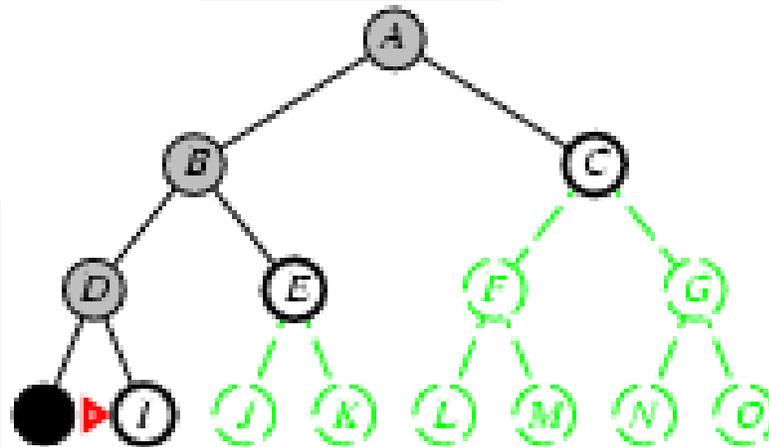
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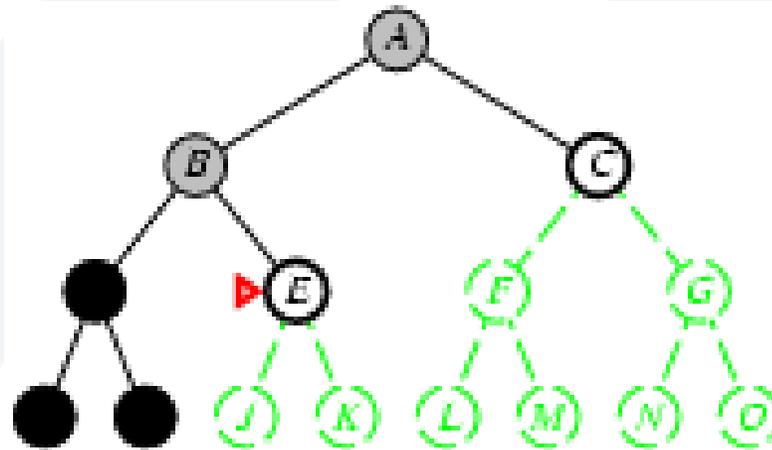
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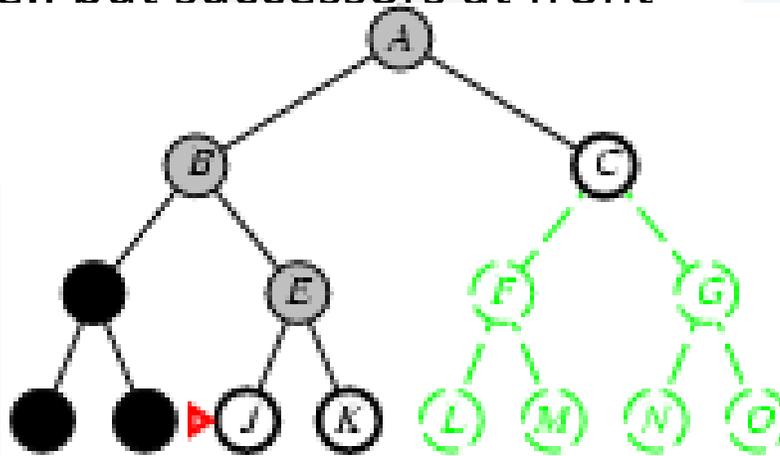
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- Expand deepest unexpanded node
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- **Implementation:**
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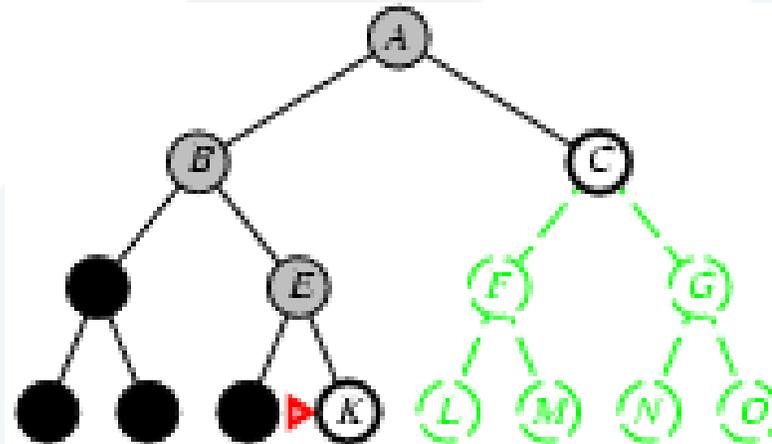
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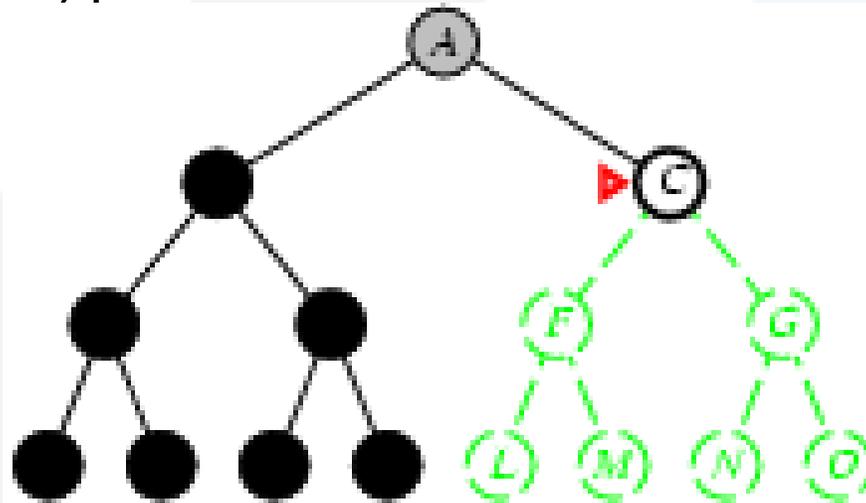
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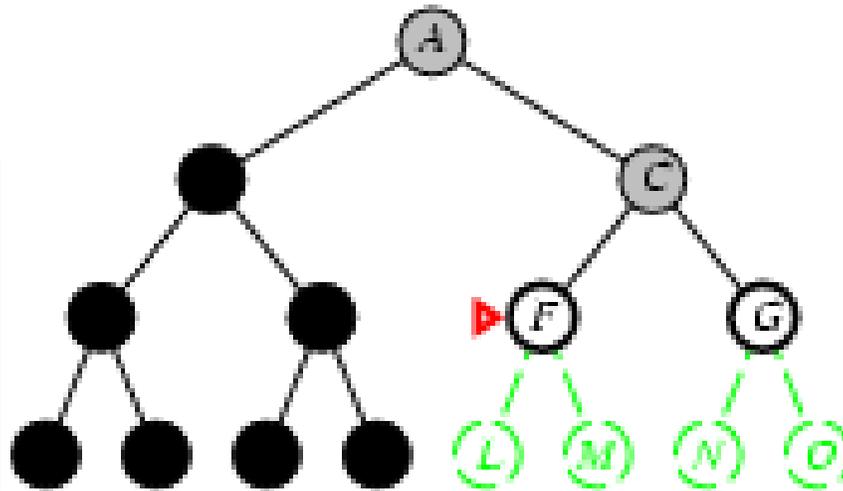
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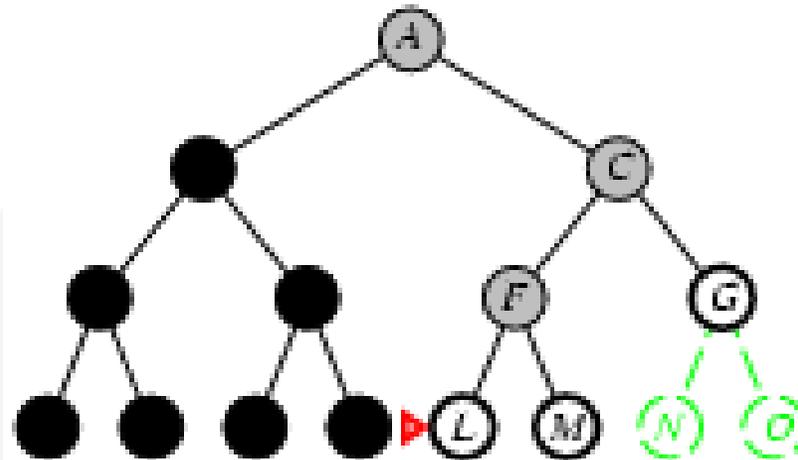
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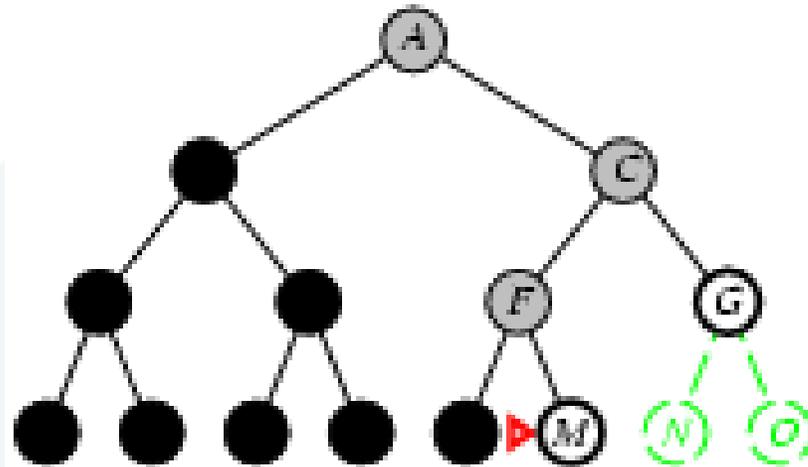
# Depth-first search

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- **Implementation:**
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  -



# Depth-first search

- Expand deepest unexpanded node
- <http://aispace.org/search/>
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front
  -



# Properties of depth-first search

- Complete? Time? Space? Optimal?
- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path (graph search)
  - - complete in finite spaces
- Time?  $O(b^m)$ : terrible if maximum depth  $m$  is much larger than solution depth  $d$ 
  - but if solutions are dense, may be much faster than breadth-first
  -
- Space?  $O(bm)$ , i.e., linear space! Store single path with unexpanded siblings.
  - Seems to be common in animals and humans.
  -
- Optimal? No.
- Important for exploration (on-line search).
-

# Depth-limited search

- depth-first search with depth limit  $l$ ,
  - i.e., nodes at depth  $l$  have no successors
  - Solves infinite loop problem
- Common AI strategy: let user choose search/resource bound.
- Complete? No if  $l < d$ :
- Time?  $O(b^l)$ :
- Space?  $O(bl)$ , i.e., linear space!
- Optimal? No if  $l > b$

# Iterative deepening search

**function** ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution, or failure

**inputs:** *problem*, a problem

**for** *depth*  $\leftarrow$  0 **to**  $\infty$  **do**

*result*  $\leftarrow$  DEPTH-LIMITED-SEARCH(*problem*, *depth*)

**if** *result*  $\neq$  cutoff **then return** *result*

# Iterative deepening search $l=0$

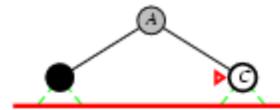
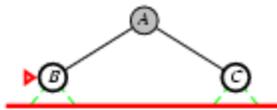
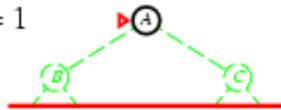
Limit = 0





# Iterative deepening search $l=1$

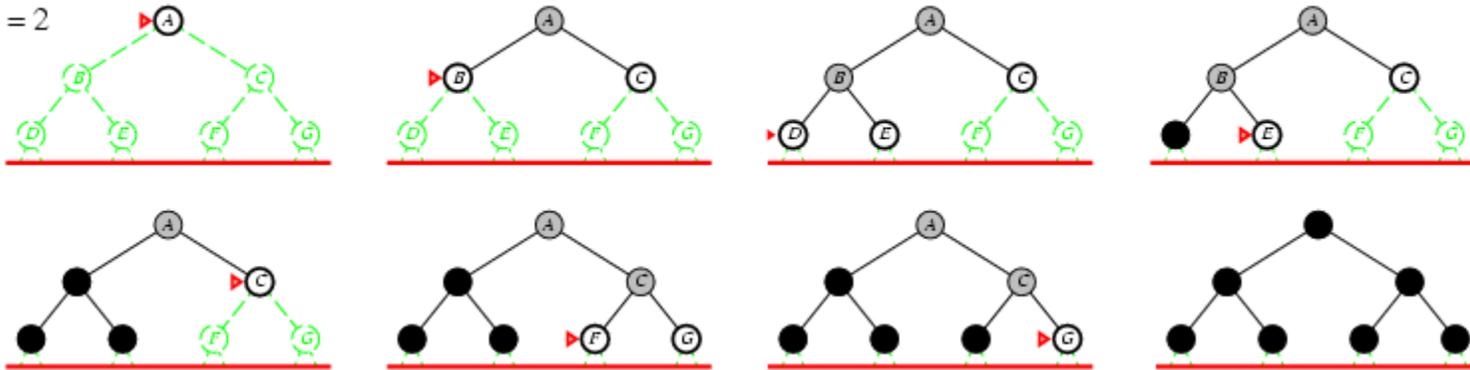
Limit = 1





# Iterative deepening search $l=2$

Limit = 2





# Iterative deepening search $l=3$

Limit = 3

