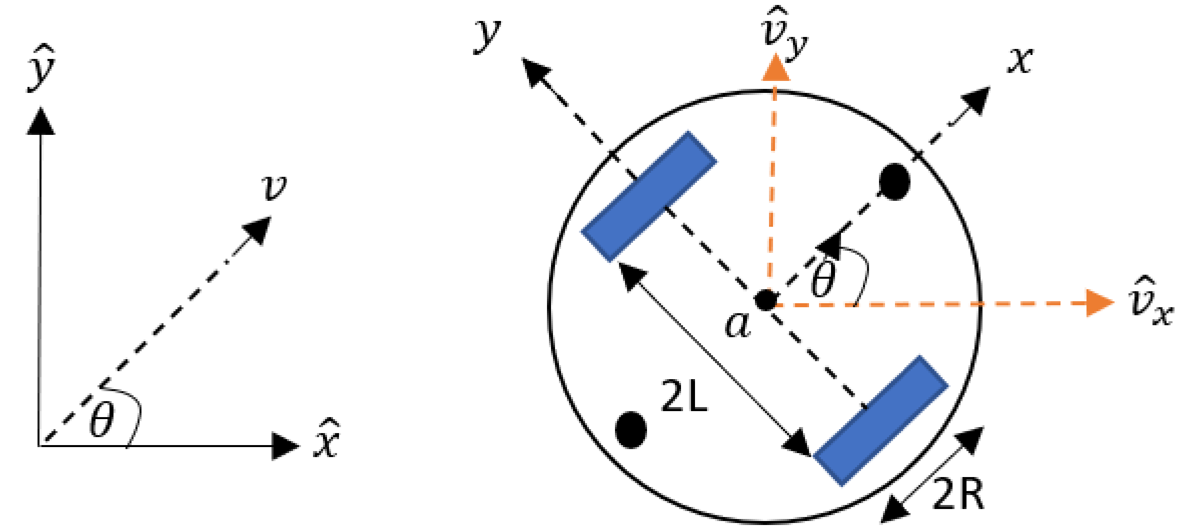
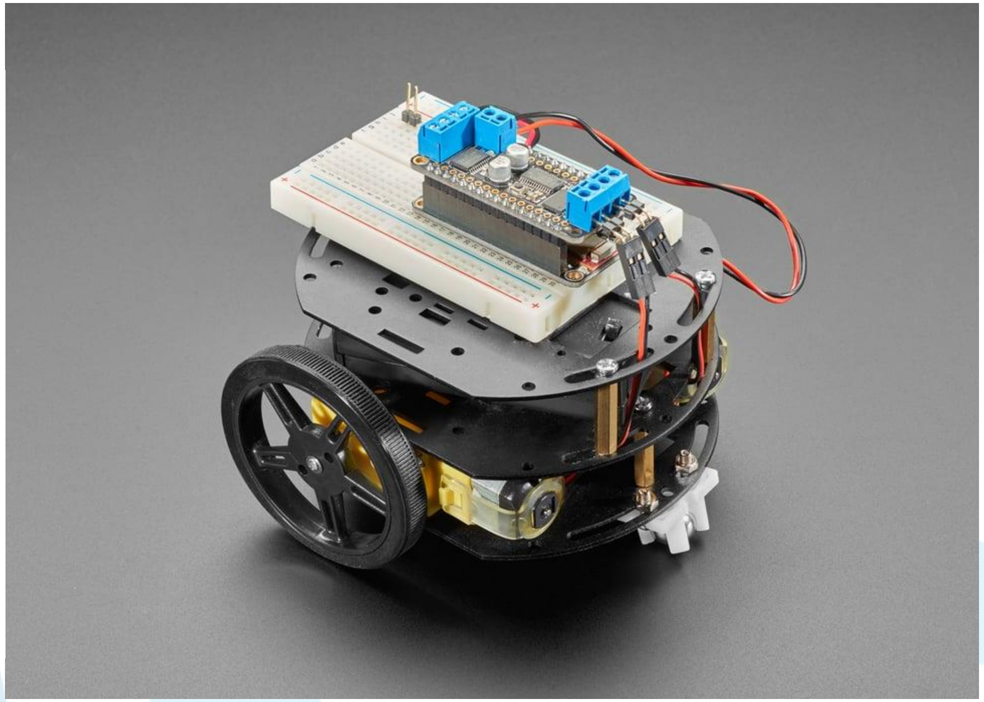
Robot Control

Motion Control of a Differential Drive Robot

Differential Drive Robot

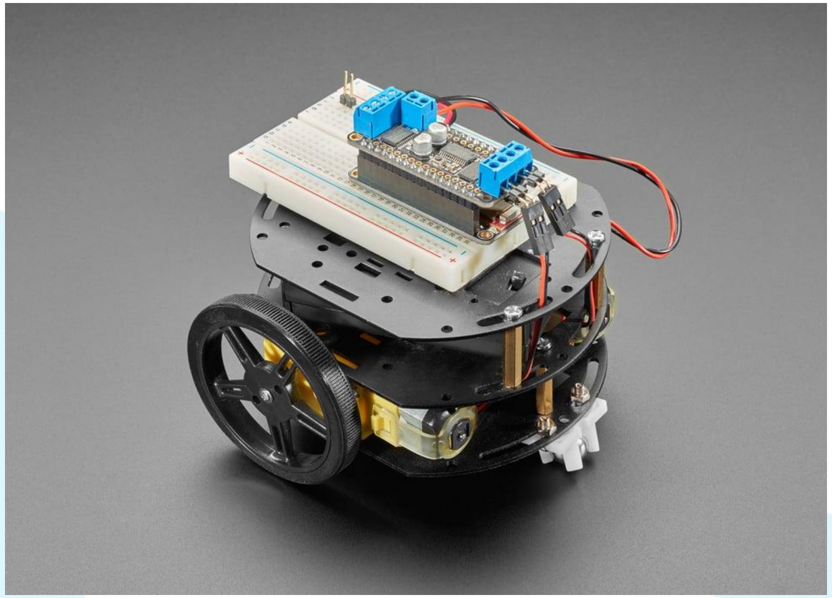




Problem

We need to move the robot from point (0,0) in the plane to the point (1,1) in 20 secs.

1. Find the equations of motion of the robot using Lagrange.
2. Compute and plot the reference trajectories of (v, w).
3. Compute and plot the required torques (τ_l, τ_r) to move the robot to the desired point.



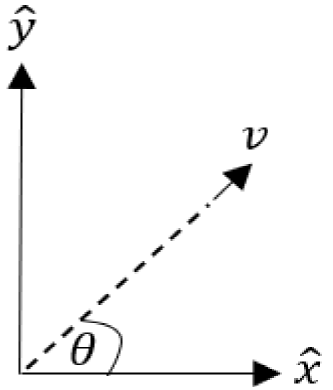
Lagrangian of the Robot

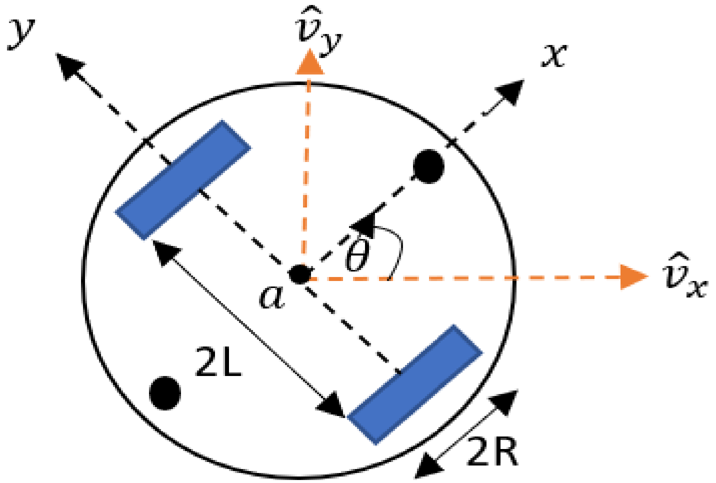
$$K_{translational} = \frac{1}{2}mv^2$$

$$K_{rotational} = \frac{1}{2}I_r\omega^2 + \frac{1}{2}I_w\omega_l^2 + \frac{1}{2}I_w\omega_r^2$$

$$V = 0$$

$$K = K_{translational} + K_{rotational}$$





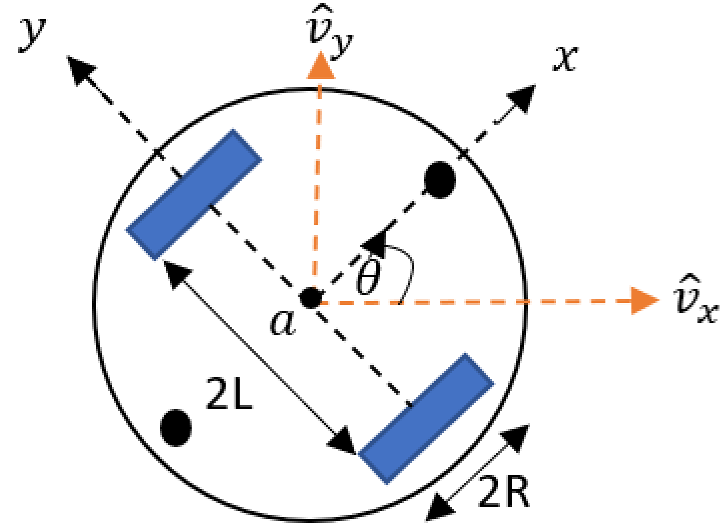
Lagrangian of the Robot

$$\mathcal{L} = K - V$$

$$\mathcal{L} = \frac{1}{2}mv^2 + \frac{1}{2}I_r\omega^2 + \frac{1}{2}I_w\omega_l^2 + \frac{1}{2}I_w\omega_r^2$$

$$v = \frac{v_r + v_l}{2} = \frac{r\dot{\theta}_r + r\dot{\theta}_l}{2}$$

$$\omega = \frac{v_r - v_l}{2L} = \frac{r\dot{\theta}_r - r\dot{\theta}_l}{2L}$$



$$\omega_r = \dot{\theta}_r$$

$$\omega_l = \dot{\theta}_l$$

Lagrangian of the Robot

$$\mathcal{L} = \frac{1}{2}m(\frac{r\dot{\theta}_r + r\dot{\theta}_l}{2})^2 + \frac{1}{2}I_r(\frac{r\dot{\theta}_r - r\dot{\theta}_l}{2L})^2 + \frac{1}{2}I_w\left(\dot{\theta}_l\right)^2 + \frac{1}{2}I_w\left(\dot{\theta}_r\right)^2$$

$$\mathcal{L} = \left(\frac{mr^2}{8} + \frac{I_r r^2}{8L^2} + \frac{1}{2}I_w\right)\left(\dot{\theta}_r\right)^2 + \left(\frac{mr^2}{8} + \frac{I_r r^2}{8L^2} + \frac{1}{2}I_w\right)\left(\dot{\theta}_l\right)^2 + (\frac{mr^2}{4} - \frac{I_r r^2}{4L^2})(\dot{\theta}_r\dot{\theta}_l)$$

Equations of motion

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_r}\right) - \left(\frac{\partial \mathcal{L}}{\partial \theta_r}\right) = \tau_r \qquad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_l}\right) - \left(\frac{\partial \mathcal{L}}{\partial \theta_l}\right) = \tau_l$$

$$\begin{pmatrix} \frac{mr^2}{4} + \frac{I_r r^2}{8L^2} + I_w & \frac{mr^2}{4} - \frac{I_r r^2}{8L^2} \\ \frac{mr^2}{4} - \frac{I_r r^2}{8L^2} & \frac{mr^2}{4} + \frac{I_r r^2}{8L^2} + I_w \end{pmatrix} \begin{pmatrix} \ddot{\theta}_r \\ \ddot{\theta}_l \end{pmatrix} = \begin{pmatrix} \tau_r \\ \tau_l \end{pmatrix}$$

Moving to the final position

To move from the starting point at the origin (0,0) to the final position at (1,1). The robot can do two consecutive movements:

1. Pure rotation from the starting orientation $\theta_s = 0$ to the final orientation θ_f determined by the final position (1,1).

2. Pure translation (straight line movement) from (0,0) to (1,1).

جَـامعة
المَـنارة
MANARA UNIVERSITY

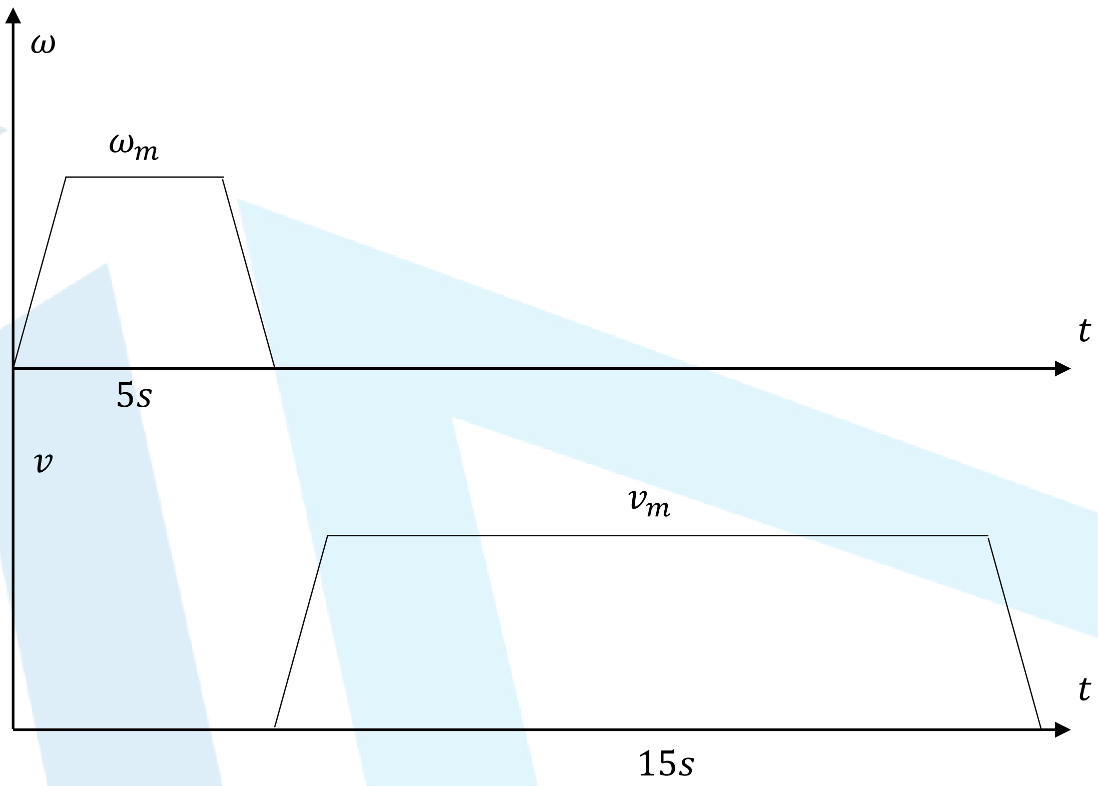
Moving to the final position

- We need now to determine the reference trajectories (v, ω).
- We use trapezoidal profiles to represent the two movements mentioned previously.
- The total time equals 20 secs, we can divide it into 5 secs for rotation and 15 secs for translation.

Moving to the final position

$$v_m = ?$$

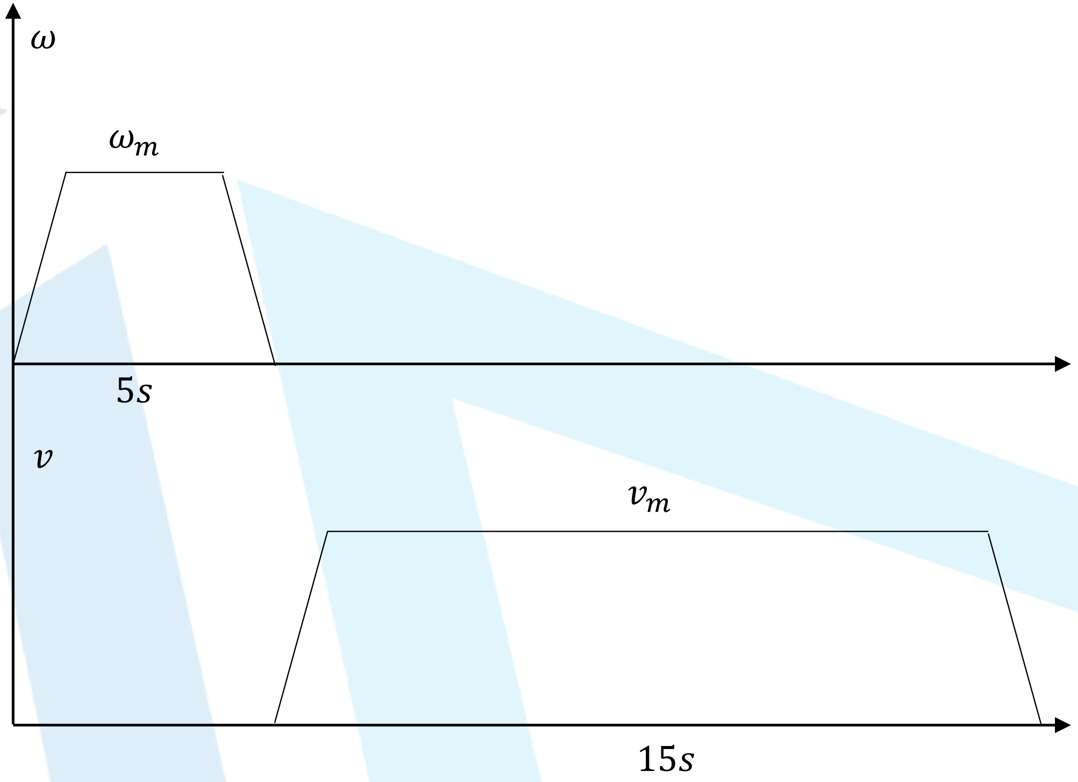
$$\omega_m = ?$$



Moving to the final position

$$v_m = \frac{2\sqrt{2}}{27}\ \mathrm{m/s}$$

$$\omega_m = \frac{\pi}{18}\ \mathrm{rad/s}$$



MATLAB Code

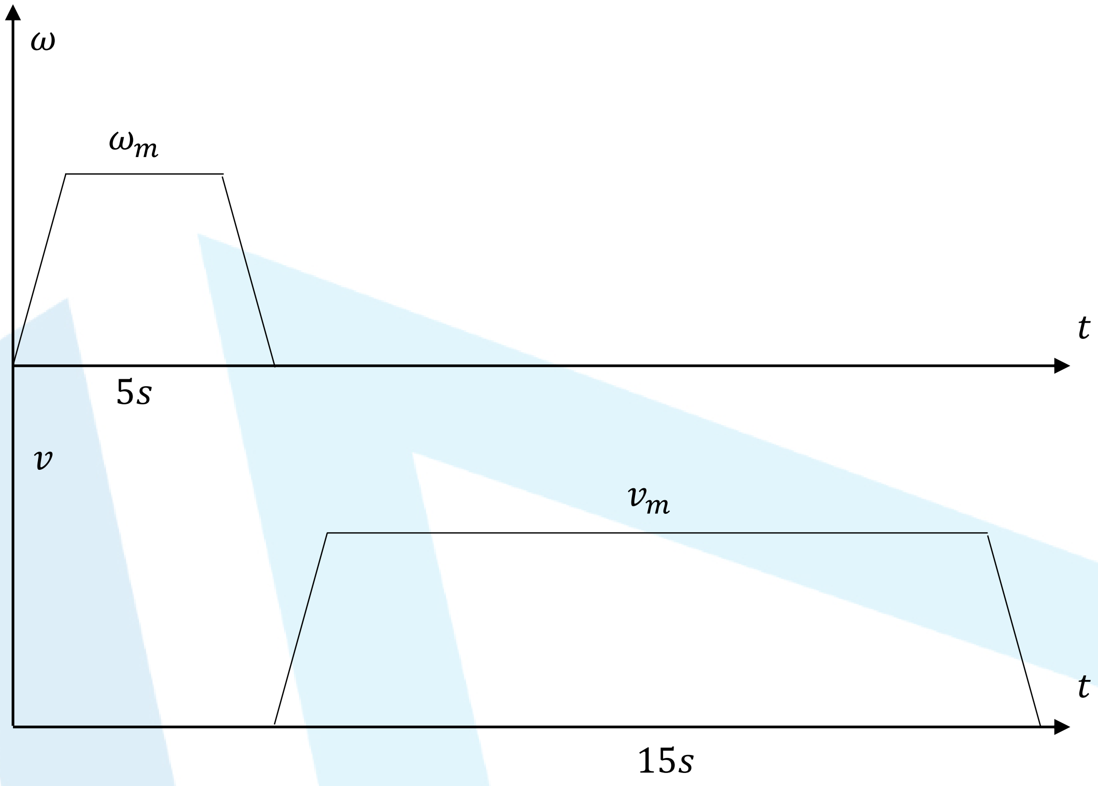
```
clc; clear;

m = 1; r = 0.08;
L = 0.1; Ir = 1; Iw = 0.1;

D = [m*r^2/4+(Ir*r^2)/(8*L^2)+Iw, m*r^2/4 - (Ir*r^2)/(8*L^2);
     m*r^2/4 - (Ir*r^2)/(8*L^2), m*r^2/4+(Ir*r^2)/(8*L^2)+Iw];
```

MATLAB Code

```
Ttotal = 20;
Tw = 5;
Tv = 15;
dt = 0.001;

vm = 2*sqrt(2)/27;
wm = pi/18;

T = 0:dt:Ttotal;
Aw = zeros(size(T));
Av = zeros(size(T));
```

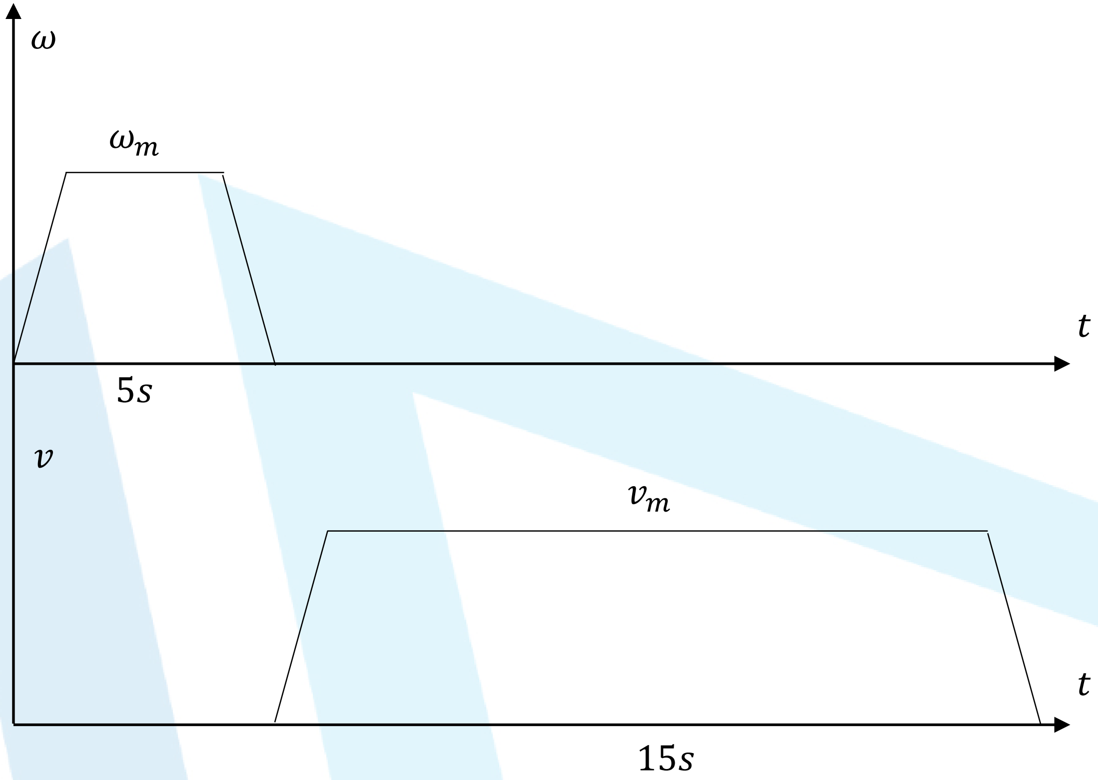
MATLAB Code

```
Aw(T > 0    & T <= 0.5) = wm/0.5;
Aw(?) = ?;
Aw(?) = ?;

Av(?) = ?;
Av(?) = ?;
Av(?) = ?;
```



MATLAB Code

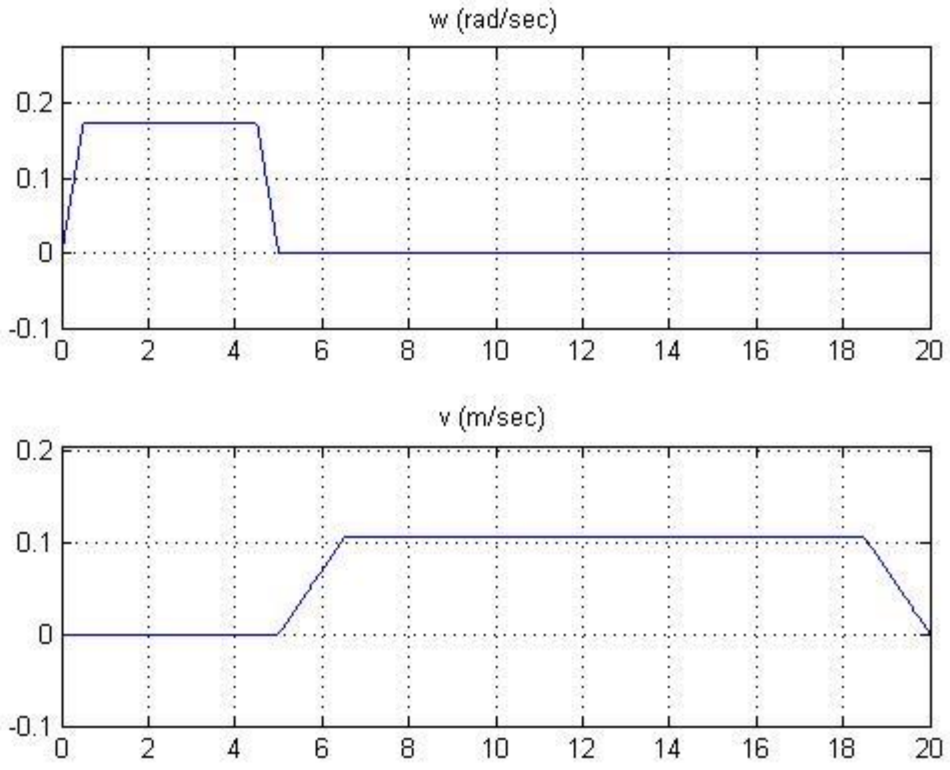
```
Aw(T > 0   & T <= 0.5) = wm/0.5;
Aw(T > 0.5 & T <= 4.5) = 0;
Aw(T > 4.5   & T <= 5) = -wm/0.5;

Av(T > 5   & T <= 6.5) = vm/1.5;
Av(T > 6.5 & T <= 18.5) = 0;
Av(T > 18.5   & T <= 20) = -vm/1.5;
```



MATLAB Code

```
W = cumtrapz(T,Aw);
V = cumtrapz(T,Av);

subplot(2,1,1)
plot(T,W), ylim([-0.1, wm+0.1])
title('w (rad/sec)'), grid

subplot(2,1,2)
plot(T,V), ylim([-0.1, vm+0.1])
title('v (m/sec)'), grid
```

MATLAB Code


w (rad/sec)
0.2
0.1
0
-0.1
0
2
4
6
8
10
12
14
16
18
20
v (m/sec)
0.2
0.1
0
-0.1
0
2
4
6
8
10
12
14
16
18
20

MATLAB Code

$$v = \frac{v_r + v_l}{2} = \frac{r\dot{\theta}_r + r\dot{\theta}_l}{2}$$

$$\dot{\theta}_r = \frac{v + \omega L}{r}$$

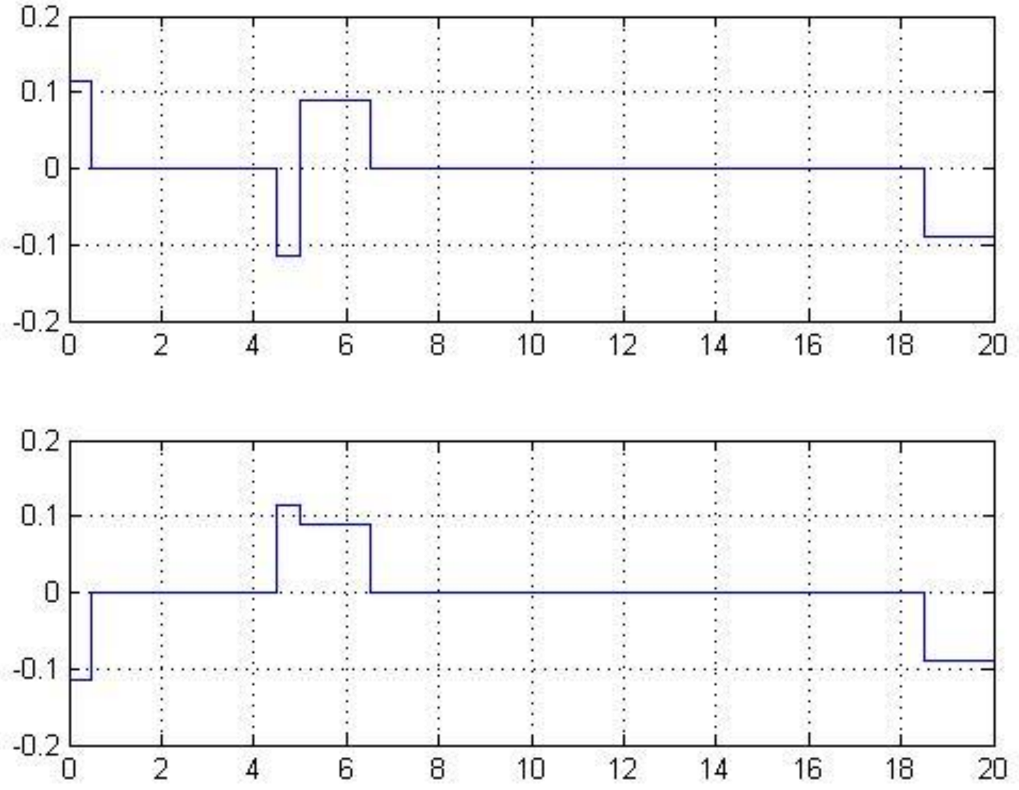
$$\omega = \frac{v_r - v_l}{2L} = \frac{r\dot{\theta}_r - r\dot{\theta}_l}{2L}$$

$$\dot{\theta}_l = \frac{v - \omega L}{r}$$

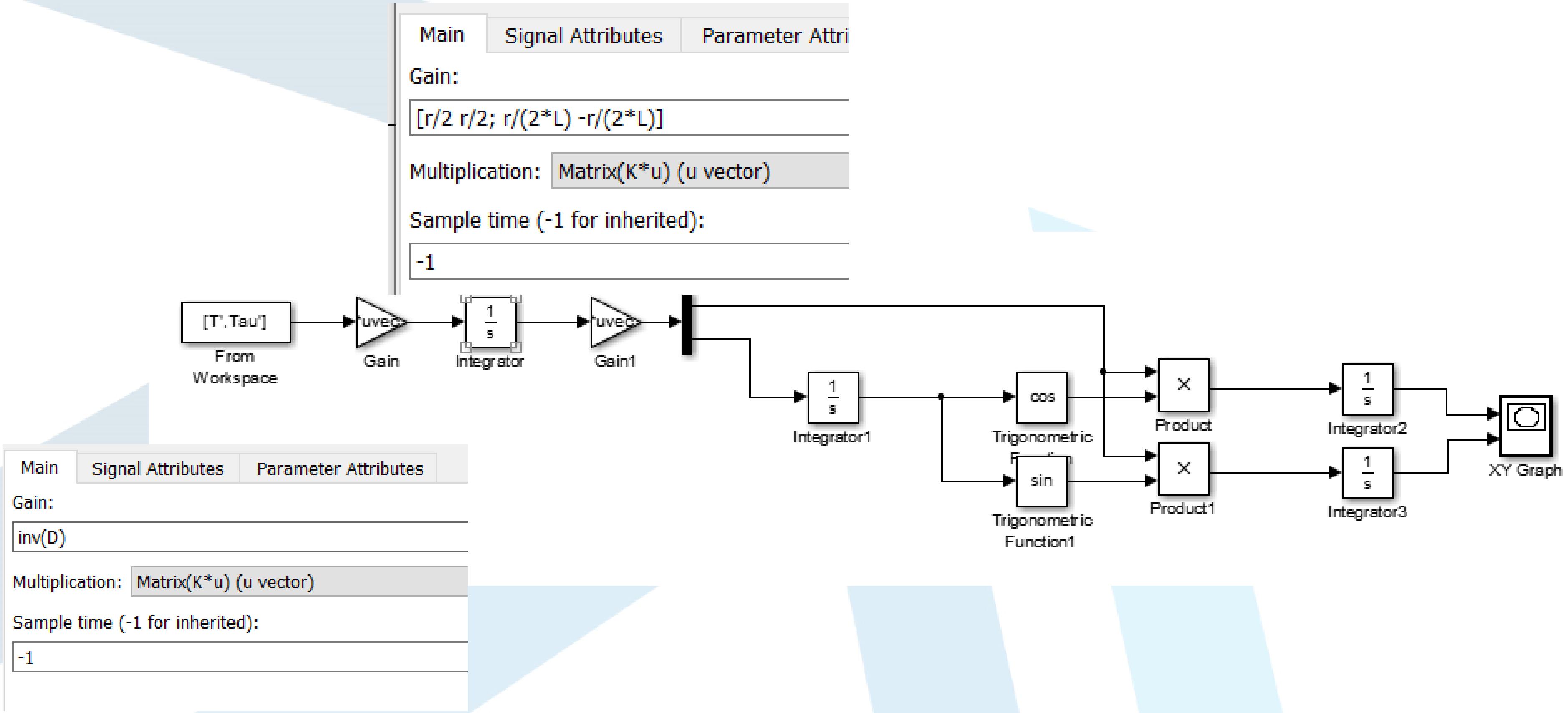
MATLAB Code

```
dThr = (V + W.*L)/r;
dThl = (V - W.*L)/r;

ddThr = [0, diff(dThr)/dt];
ddThl = [0, diff(dThl)/dt];

ddTh = [ddThr; ddThl];
Tau = D*ddTh;

close all
subplot(2,1,1), plot(T,Tau(1,:)), grid
subplot(2,1,2), plot(T,Tau(2,:)), grid
```

MATLAB Code


0.2
0.1
0
-0.1
-0.2
0 2 4 6 8 10 12 14 16 18 20
0.2
0.1
0
-0.1
-0.2
0 2 4 6 8 10 12 14 16 18 20

Animation (Simulink)



Thanks